Question Number	Scheme	Notes	Marks
5(a)	$4x^2+4x+17=4\left(x^2+x+\frac{17}{4}\right)=4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+\frac{17}{4}\right]=(2x+1)^2+16$ or $4x^2+4x+17=4x^2+4px+q \Rightarrow 4px=4x \Rightarrow \underline{p=1}$, $q+p^2=17 \Rightarrow \underline{q=16}$ B1: Either p or q correct B1: Both correct values in part (a). Allow from any/no work. Values may be embedded within expression $(2x+p)^2+q$.		**B1** **B1**
			(2)
(b)	$A=8,\ B=4$	Both correct values (accept if embedded)	**B1**
			(1)
(c)	Note that this is a Hence question and there is no credit for work on the original fraction		
	$\int\frac{8x+5}{\sqrt{4x^2+4x+17}}\,dx=\int\frac{1}{\sqrt{(2x+1)^2+16}}\,dx+\int\frac{8x+4}{\sqrt{4x^2+4x+17}}\,dx$ $=\frac{1}{2}\text{arsinh}\left(\frac{2x+1}{4}\right)+2\left(4x^2+4x+17\right)^{\frac{1}{2}}$ or $\frac{1}{2}\ln\left(\frac{2x+1}{4}+\sqrt{\left(\frac{2x+1}{4}\right)^2+1}\right)+2\left(4x^2+4x+17\right)^{\frac{1}{2}}$ or $\frac{1}{2}\ln\left(2x+1+\sqrt{(2x+1)^2+16}\right)+2\left((2x+1)^2+16\right)^{\frac{1}{2}}$	M1: For $\ldots\text{arsinh}(f(x))$ $f(x)\neq k$ or logarithmic equivalent i.e., $\ldots\ln\left(f(x)+\sqrt{(f(x))^2+c}\right)$ $c\neq 0$ M1: For $\ldots\left(4x^2+4x+17\right)^{\frac{1}{2}}$ or $\ldots\left((2x+1)^2+16\right)^{\frac{1}{2}}$ A1: Fully correct integration	**M1** **M1** **A1**
	Allow for equivalents in e.g., u if substitutions are used e.g., $u=2x+1\Rightarrow\frac{1}{2}\int\frac{1}{\sqrt{u^2+16}}\,du\Rightarrow\frac{1}{2}\text{arsinh}\left(\frac{u}{4}\right)$ $u=4x^2+4x+17\Rightarrow\int\frac{1}{\sqrt{u}}\,du\Rightarrow 2\sqrt{u}$ $4\sinh u=2x+1\Rightarrow\int\frac{2\cosh u}{\sqrt{16\cosh^2 u}}\,du\Rightarrow\frac{1}{2}\int du=\frac{1}{2}u$ Score the M marks for appropriate forms (sign/coefficient errors only). If they continue working in terms of u the limits applied for the **dd**M1 must be correct for their substitution which for the above examples would be 3 & $\frac{5}{3}$, 25 & $\frac{169}{9}$ and $\text{arsinh}\left(\frac{3}{4}\right)$ & $\text{arsinh}\left(\frac{5}{12}\right)$		
	$\int_{\frac{1}{3}}^{1}\frac{8x+5}{\sqrt{4x^2+4x+17}}\,dx=\frac{1}{2}\text{arsinh}\left(\frac{3}{4}\right)-\frac{1}{2}\text{arsinh}\left(\frac{5}{12}\right)+2\sqrt{25}-2\sqrt{\frac{169}{9}}$ $\Rightarrow\frac{1}{2}\ln\left(\frac{3}{4}+\sqrt{\left(\frac{3}{4}\right)^2+1}\right)-\frac{1}{2}\ln\left(\frac{5}{12}+\sqrt{\left(\frac{5}{12}\right)^2+1}\right)+2\sqrt{25}-\frac{26}{3}$ Condone replacement of $\text{arsinh}\left(\frac{x}{a}\right)$ with $\ln\left(x+\sqrt{x^2+a^2}\right)$ with $a\neq 1$ instead of using $\text{arsinh}x=\ln\left(x+\sqrt{x^2+1}\right)$	Substitutes and subtracts with the given limits and uses the appropriate form for arsinh twice (if required). Results from separate integrals must be combined. Allow slips but the $f\left(\frac{1}{3}\right)$ terms (and no others) must be subtracted. Not implied by just the final answer. **Requires both previous M marks.**	**ddM1**
	arsinh(...) may be evaluated using correct exp definition & solving a exponential 3TQ		
	$\left\{=\frac{1}{2}\ln 2-\frac{1}{2}\ln\frac{3}{2}+10-\frac{26}{3}\right\}=\frac{4}{3}+\frac{1}{2}\ln\frac{4}{3}$ Correct answer in correct form. May be no further work following substitution but there must be nothing incorrect. Allow $k=\frac{4}{3}$ if $k+\frac{1}{2}\ln k$ is seen. Allow $\frac{1}{2}\ln\frac{4}{3}+\frac{4}{3}$		**A1**
	Algebraic integration must be used. Answer or 1.47717... only scores no marks		
			(5)
			Total 8