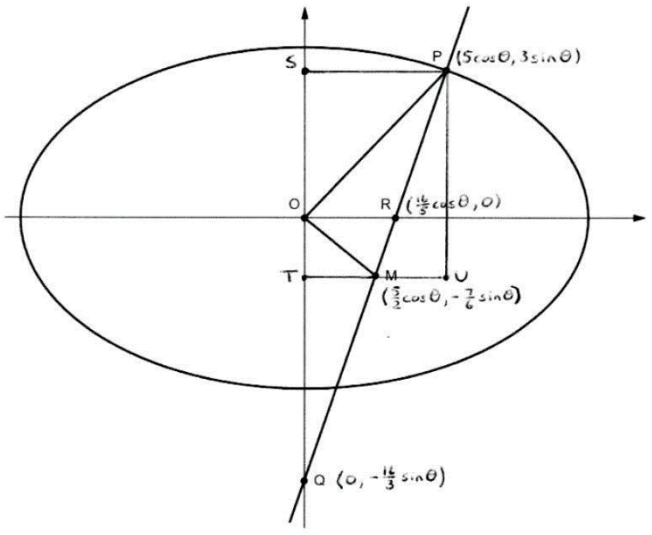
Question Number	Scheme	Notes	Marks
6(a)	$\frac{x^2}{25}+\frac{y^2}{9}=1 \qquad P\left(5\cos\theta,\ 3\sin\theta\right)$		
	$\left\{\frac{\mathrm{d}x}{\mathrm{d}\theta}=-5\sin\theta \quad \frac{\mathrm{d}y}{\mathrm{d}\theta}=3\cos\theta\right\}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3\cos\theta}{5\sin\theta}$	$\frac{2x}{25}+\frac{2y}{9}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{9x}{25y}\left\{=-\frac{45\cos\theta}{75\sin\theta}\right\}$	$y=\left(9-\frac{9}{25}x^2\right)^{\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\frac{18}{25}x}{2\sqrt{9-\frac{9}{25}x^2}}\left\{=\frac{-\frac{18}{25}\times5\cos\theta}{2\sqrt{9-9\cos^2\theta}}\right\}$	**B1**
	Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ, or x and y, or x. Allow for a correct $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}$		
	$m_{\mathrm{T}}=-\frac{3\cos\theta}{5\sin\theta}\Rightarrow m_{\mathrm{N}}=\frac{5\sin\theta}{3\cos\theta}$	Correct perpendicular gradient rule for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ **in terms of** θ May see $m_{\mathrm{T}}=-\frac{3}{5}\cot\theta\Rightarrow m_{\mathrm{N}}=\frac{5}{3}\tan\theta$	**M1**
	$y-3\sin\theta=\frac{5\sin\theta}{3\cos\theta}\left(x-5\cos\theta\right)$ OR $y=mx+c\Rightarrow 3\sin\theta=\frac{5\sin\theta}{3\cos\theta}\times5\cos\theta+c\Rightarrow c=-\frac{16}{3}\sin\theta$ Correct straight line method with a changed gradient in terms of θ		**M1**
	$3y\cos\theta-9\sin\theta\cos\theta=5x\sin\theta-25\sin\theta\cos\theta$ $\Rightarrow 5x\sin\theta-3y\cos\theta=16\sin\theta\cos\theta$ *	Reaches given answer with intermediate line of working and no errors. Allow this equation written in reverse, x and y terms in different order provided they are together with the third term on the other side and allow the products in a different order provided the numerical coefficients "5", "–3" and "16" are at the front of the terms.	**A1***
	The last three marks require $P\left(5\cos\theta,\ 3\sin\theta\right)$ to be substituted but condone using e.g, $\frac{25y}{9x}$ as the normal gradient when forming the straight line <u>provided</u> appropriate substitution is seen before the given answer.		
			(4)

Question Number	Scheme	Notes	Marks
6(b)	At Q, $x=0 \Rightarrow y=-\frac{16}{3}\sin\theta$	**Correct** y coordinate of Q. Accept unsimplified	**B1**
	M is $\left(\frac{5\cos\theta+0}{2},\ \frac{3\sin\theta+'-\frac{16}{3}\sin\theta'}{2}\right)$ Accept $x=\frac{5}{2}\cos\theta,\ y=-\frac{7}{6}\sin\theta$	Correct method for midpoint for both coordinates with their y_Q. Could be implied. **Alternatively**, award for $\Delta OPM=\frac{1}{2}\Delta OPQ=\frac{1}{2}\times\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta$ (see area examples below)	**M1**
	e.g., PQ meets x-axis at $R\left(\frac{16}{5}\cos\theta,\ 0\right)$ $\Rightarrow$ Area $\Delta OPM=\Delta OPR+\Delta OMR$ $=\frac{1}{2}\times\frac{16}{5}\cos\theta\left(3\sin\theta+\frac{7}{6}\sin\theta\right)$	Correct unsimplified expression for area of ΔOPM Do not allow recovery from a negative area. **Can only follow incorrect work** i.e., **an incorrect midpoint if** $\Delta OPM=\frac{1}{2}\Delta OPQ$ **is used.** Please see below for alternatives	**M1**
	If shoelace method is used, score for a correct "extracted" expression for the area (allow with modulus if correct) e.g., $\frac{1}{2}\begin{vmatrix}0 & 5\cos\theta & \frac{5}{2}\cos\theta & 0\\ 0 & 3\sin\theta & -\frac{7}{6}\sin\theta & 0\end{vmatrix}$ $\Rightarrow\frac{1}{2}\left\lvert(5\cos\theta)\left(-\frac{7}{6}\sin\theta\right)-\left(\frac{5}{2}\cos\theta\right)(3\sin\theta)\right\rvert$ or $\frac{1}{2}\left[(5\cos\theta)\left(\frac{7}{6}\sin\theta\right)+\left(\frac{5}{2}\cos\theta\right)(3\sin\theta)\right]$		
	$\left\{=\frac{20}{3}\sin\theta\cos\theta=\frac{10}{3}\sin2\theta\right\}\Rightarrow(\text{area}=)\frac{10}{3}$	Correct area <u>following a correct expression</u>	**A1**
	$\frac{10}{3}$ <u>and justification</u>: **From** $\frac{10}{3}\sin2\theta$: max (value) of $\sin2\theta$ is 1 or e.g., $-1\leqslant\sin2\theta\leqslant1$ or states $\theta=\frac{\pi}{4}$ or 45° or obtains this using differentiation: $\left\{\frac{10}{3}\right\}\sin2\theta\Rightarrow\left\{\frac{20}{3}\right\}\cos2\theta=0\Rightarrow\ldots$ Do not accept if there is any wrong statement e.g., $\sin2\theta\leqslant1$, $-1<\sin2\theta<1$ but we will condone the ambiguous "$\sin2\theta$ is between 1 and -1" **From any other expression**: Must <u>differentiate</u> (unless rewrites as $\frac{10}{3}\sin2\theta$) e.g., $\frac{20}{3}\sin\theta\cos\theta$ $\frac{20}{3}\left(\cos^2\theta-\sin^2\theta\right)\Rightarrow\frac{20}{3}\cos2\theta=0$ or $\tan^2\theta=1\Rightarrow\theta=\frac{\pi}{4}$ or 45° Ignore any further differentiation to justify maximum		**A1**
			(5)
			Total 9



May see:

$\Delta OPM=\frac{1}{2}\Delta OPQ=\frac{1}{2}\times\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta$

(Scores the first 2 M marks together since M is not required – ignore an absent or wrong M)

$\Delta OPM=\Delta OPQ-\Delta OMQ$

$\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta-\frac{1}{2}\times\frac{16}{3}\sin\theta\times\frac{5}{2}\cos\theta$

$\Delta OPM=\Delta PQS-\Delta OMQ-\Delta PSO$

$=\frac{1}{2}\times\left(\frac{16}{3}\sin\theta+3\sin\theta\right)\times5\cos\theta-\frac{1}{2}\times\frac{16}{3}\sin\theta\times\frac{5}{2}\cos\theta-\frac{1}{2}\times3\sin\theta\times5\cos\theta$

$\left\{=\frac{125}{6}\sin\theta\cos\theta-\frac{20}{3}\sin\theta\cos\theta-\frac{15}{2}\sin\theta\cos\theta\right\}$

$\Delta OPM=PSTU-\Delta PSO-\Delta OMT-\Delta PMU$

$=5\cos\theta\times\left(3\sin\theta+\frac{7}{6}\sin\theta\right)-\frac{1}{2}\times3\sin\theta\times5\cos\theta$

$-\frac{1}{2}\times\frac{5}{2}\cos\theta\times\frac{7}{6}\sin\theta-\frac{1}{2}\times\left(5\cos\theta-\frac{5}{2}\cos\theta\right)\left(3\sin\theta+\frac{7}{6}\sin\theta\right)$

$\left\{=\left(\frac{125}{6}-\frac{15}{2}-\frac{35}{24}-\frac{125}{24}\right)\sin\theta\cos\theta\right\}$

Note that attempts that start by using Pythagoras for PM will also require the perpendicular distance from O to the line