Question Number	Scheme	Notes	Marks
7	$y = \ln\left(\tanh\frac{x}{2}\right) \qquad 1 \leqslant x \leqslant 2$		
(a)	$\frac{dy}{dx} = \frac{1}{\tanh\frac{x}{2}} \times \frac{1}{2}\text{sech}^2\frac{x}{2}$ or e.g., $\frac{1}{2}\coth\frac{x}{2}\left(1-\tanh^2\frac{x}{2}\right)$ or $e^y = \tanh\frac{x}{2} \Rightarrow \left(\tanh\frac{x}{2}\right)\frac{dy}{dx} = \frac{1}{2}\text{sech}^2\frac{x}{2}$ or $\Rightarrow \text{artanh}\left(e^y\right) = \frac{x}{2} \Rightarrow \left(\frac{e^y}{1-e^{2y}}\right)\frac{dy}{dx} = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\coth\frac{x}{2}\left(1-\tanh^2\left(\frac{x}{2}\right)\right)$ Obtains an expression for (or equation involving) $\frac{dy}{dx}$ of appropriate form. Condone sign/coefficient errors only and any $\frac{x}{2}$s written as x but no "y"s. Do not condone missing "h"s in hyperbolic functions unless they are recovered		**M1**
	$\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \Rightarrow \int\sqrt{1+\left(\frac{\text{sech}^2\frac{x}{2}}{2\tanh\frac{x}{2}}\right)^2}(dx)$ or e.g., $\sqrt{1+\left(\frac{dy}{dx}\right)^2} \Rightarrow \sqrt{1+\left(\frac{\cosh\frac{x}{2}}{2\sinh\frac{x}{2}\cosh^2\frac{x}{2}}\right)^2}$ Applies arc length formula (with or without the integration sign) with their $\frac{dy}{dx}$ which may have been simplified incorrectly before substitution. Do not condone attempts that clearly have worked backwards to deduce that the derivative is cosech x. Also condone incorrect work processing $1+\left(\frac{dy}{dx}\right)^2$ provided the expression is shown as square rooted afterwards. Not dependent. Ignore any multiplier such as π or 2π or k but forming $y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is M0		**M1**
	$\sqrt{1+\left(\frac{1}{2\sinh\frac{x}{2}\cosh\frac{x}{2}}\right)^2} \rightarrow \sqrt{1+\left(\frac{1}{\sinh x}\right)^2}$ Uses identity/identities (sign errors only) to obtain $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ in terms of x and not $\frac{x}{2}$. Attempts that square the derivative and add the 1 first before attempting to convert to x must be convincing. **Requires both previous M marks.**		**ddM1**
	$\sqrt{1+\left(\frac{1}{\sinh x}\right)^2} = \sqrt{1+\text{cosech}^2 x} \Rightarrow s = \int_1^2 \coth x\,dx$ or e.g., $= \int\sqrt{\frac{\sinh^2 x+1}{\sinh^2 x}}\,dx \Rightarrow s = \int_1^2 \coth x\,dx$ Obtains given answer with no errors and at least one non-trivial intermediate line following the first expression in terms of x. Allow without "$s =$" but RHS must be exactly as printed. Allow a "meet in the middle" approach but it must be convincing and $\int_1^2 \coth x\,dx$ must be seen. No missing "h"s in hyperbolic functions or missing arguments even if recovered. We will condone the occasional notational error e.g., $\text{sech}\frac{x}{2}^2$ if recovered		**A1***
			(4)

Question Number	Scheme	Notes	Marks
7(b)	$\int \coth x \,\mathrm{d}x = \ln\left(\sinh x\right)$ Correct integration. May see $-\ln\left(\text{cosech}\, x\right)$ May see the sinh x in exponentials without the "2" which may come from the substitution $u = \mathrm{e}^{x} - \mathrm{e}^{-x}$ i.e., $\ln\left(\mathrm{e}^{x} - \mathrm{e}^{-x}\right)$		**B1**
	1, 2 & 3. $\ln\left(\dfrac{\mathrm{e}^2 - \mathrm{e}^{-2}}{2}\right) - \ln\left(\dfrac{\mathrm{e} - \mathrm{e}^{-1}}{2}\right) = \ln\left(\dfrac{\mathrm{e}^2 - \dfrac{1}{\mathrm{e}^2}}{\mathrm{e} - \dfrac{1}{\mathrm{e}}}\right)$ or **4.** $\ln\left(\sinh 2\right) - \ln\left(\sinh 1\right) = \ln\left(\dfrac{\sinh 2}{\sinh 1}\right)$ Following replacement of $\int \coth x \,\mathrm{d}x$ with $\pm\ln\left(\sinh x\right), \pm\ln\left(\cosh x\right), \pm\ln\left(\text{cosech}\, x\right)$ or $\pm\ln\left(\text{sech}\, x\right)$, substitutes given limits, subtracts and writes as a single logarithm. Condone sign errors if exponential forms used and may use negative powers of e.		**M1**
	1. $\ln\left(\dfrac{\mathrm{e}^2 - \dfrac{1}{\mathrm{e}^2}}{\mathrm{e} - \dfrac{1}{\mathrm{e}}}\right) = \ln\left(\dfrac{\dfrac{\mathrm{e}^4 - 1}{\mathrm{e}^2}}{\dfrac{\mathrm{e}^2 - 1}{\mathrm{e}}}\right) = \ln\left(\dfrac{\mathrm{e}^4 - 1}{\mathrm{e}^3 - \mathrm{e}}\right)$ or **2.** $\Rightarrow \ln\left(\dfrac{\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)}{\mathrm{e} - \dfrac{1}{\mathrm{e}}}\right)$ or $\ln\left(\dfrac{\left(\mathrm{e} + \mathrm{e}^{-1}\right)\left(\mathrm{e} - \mathrm{e}^{-1}\right)}{\left(\mathrm{e} - \mathrm{e}^{-1}\right)}\right)$ or **3.** $\Rightarrow \ln\left(\dfrac{\mathrm{e}^2 - \mathrm{e}^{-2}}{\mathrm{e} - \mathrm{e}^{-1}} \times \dfrac{\mathrm{e} + \mathrm{e}^{-1}}{\mathrm{e} + \mathrm{e}^{-1}}\right) = \ln\left(\dfrac{\left(\mathrm{e}^2 - \mathrm{e}^{-2}\right)\left(\mathrm{e} + \mathrm{e}^{-1}\right)}{\mathrm{e}^2 - \mathrm{e}^{-2}}\right)$ or **4.** $\ln\left(\dfrac{\sinh 2}{\sinh 1}\right) = \ln\left(\dfrac{2\sinh 1 \cosh 1}{\sinh 1}\right)$ Following use of correct exponential form for sinh/cosech: **1.** Obtains a **correct** ln of a **single** fraction (or product of **single** fractions) with no negative powers of e **or** **2.** Uses difference of two squares to correctly factorise numerator **or** **3.** Applies correct multiplier to achieve expression shown **or** **4.** Correctly replaces $\sinh 2$ with $2\sinh 1\cosh 1$ allowing equivalent work e.g., $\dfrac{\sinh 2}{\sinh 1} = \sqrt{\dfrac{\left(2\cosh^2 1 - 1\right)^2 - 1}{\cosh^2 1 - 1}} = \sqrt{\dfrac{4\cosh^4 1 - 4\cosh^2 1}{\cosh^2 1 - 1}} \Rightarrow s = \ln\sqrt{4\cosh^2 1}$ **Requires previous M mark.**		**dM1**
	1. $s = \ln\left(\dfrac{\left(\mathrm{e}^2 + 1\right)\left(\mathrm{e}^2 - 1\right)}{\mathrm{e}\left(\mathrm{e}^2 - 1\right)}\right) = \ln\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)$ or **2 & 3.** $s = \ln\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)$ or **4.** $s = \ln\left(2\cosh 1\right)$ or $\ln\left(2\left(\dfrac{\mathrm{e} + \mathrm{e}^{-1}}{2}\right)\right) = \ln\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)$	Obtains given answer from complete and correct work. Minimum for each route shown. Allow $\ln\left(\mathrm{e}^{-1} + \mathrm{e}\right)$	**A1***
	Algebraic integration must be used		
	Note that there are potentially other ways e.g., factorising followed by log laws: $\ln\left(\dfrac{\mathrm{e}^2 - \mathrm{e}^{-2}}{2}\right) - \ln\left(\dfrac{\mathrm{e} - \mathrm{e}^{-1}}{2}\right) = \ln\left(\dfrac{1}{2}\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)\right) - \ln\left(\dfrac{1}{2}\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)\right)$ M1 $= \ln\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right) + \ln\left(\dfrac{1}{2}\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)\right) - \ln\left(\dfrac{1}{2}\left(\mathrm{e} - \dfrac{1}{\mathrm{e}}\right)\right)$ dM1 $= \ln\left(\mathrm{e} + \dfrac{1}{\mathrm{e}}\right)$ A1*		
			(4)
			Total 8