Question Number	Scheme	Notes	Marks
8	$I_n = \int_0^k x^n \left(k-x\right)^{\frac{1}{2}} \mathrm{d}x \qquad n \geqslant 0$ If d(...) notation is used marks are only scored when it is removed. **Please see overleaf if the split is done first**		
(a)	$u = x^n \qquad u' = nx^{n-1} \qquad v' = \left(k-x\right)^{\frac{1}{2}} \qquad v = -\frac{2}{3}\left(k-x\right)^{\frac{3}{2}}$ $I_n = \left[-\frac{2}{3}x^n\left(k-x\right)^{\frac{3}{2}}\right]_0^k - \int_0^k -\frac{2}{3}nx^{n-1}\left(k-x\right)^{\frac{3}{2}}\mathrm{d}x$ M1: Uses parts in the correct direction to obtain an expression of the form $\pm ...x^n\left(k-x\right)^{\frac{3}{2}} \pm \int ...x^{n-1}\left(k-x\right)^{\frac{3}{2}}\left(\mathrm{d}x\right)$ A1: Correct expression (limits not required on either part and 'dx' may be missing)		**M1** **A1**
	$\left(I_n =\right)\ 0+\frac{2}{3}n\int_0^k x^{n-1}\left(k-x\right)\left(k-x\right)^{\frac{1}{2}}\mathrm{d}x$	Applies $\left(k-x\right)^{\frac{3}{2}} = \left(k-x\right)\left(k-x\right)^{\frac{1}{2}}$ to integral. Could be implied if work correct but do not accept going straight to $"\frac{2}{3}nkI_{n-1} - \frac{2}{3}nI_n"$ **Requires previous M mark.**	**dM1**
	$\frac{2}{3}n\int_0^k\left(kx^{n-1}\left(k-x\right)^{\frac{1}{2}} - x^n\left(k-x\right)^{\frac{1}{2}}\right)\mathrm{d}x$ $\Rightarrow \frac{2}{3}n\left(kI_{n-1}-I_n\right)$ or $\frac{2}{3}knI_{n-1}-\frac{2}{3}nI_n$ or $\frac{2}{3}kn\int_0^k x^{n-1}\left(k-x\right)^{\frac{1}{2}}\left(\mathrm{d}x\right) - \frac{2}{3}n\int_0^k x^n\left(k-x\right)^{\frac{1}{2}}\left(\mathrm{d}x\right)$	Expands and writes RHS in terms of both I_n and I_{n-1} i.e., RHS = $..I_{n-1} \pm ...I_n$ with no other terms. This mark is not available until the $\left[...x^n\left(k-x\right)^{\frac{3}{2}}\right]_0^k$ disappears. Allow if actual integrals are used for both I_n and/or I_{n-1} and allow going straight to $\frac{2}{3}knI_{n-1}-\frac{2}{3}nI_n$ provided the split was seen. **Requires both previous M marks.**	**ddM1**
	$\Rightarrow \left(1+\frac{2}{3}n\right)I_n = \frac{2}{3}knI_{n-1}$ or $\Rightarrow \frac{3+2n}{3}I_n = \frac{2}{3}knI_{n-1}$ $\Rightarrow I_n = \frac{2kn}{3+2n}I_{n-1}$* Reaches given answer with no mathematical errors seen. Allow poor bracketing if it is recovered. There must be at least one non-trivial intermediate line where the LHS $= \mathrm{f}\left(n\right)I_n$ allowing e.g., $I_n + \frac{2}{3}I_n = ...$ Allow minor variations in given answer e.g., $I_n = \frac{2nkI_{n-1}}{2n+3}$ Condone missing 'dx's and allow if limits only seen once but $\left[-\frac{2}{3}x^n\left(k-x\right)^{\frac{3}{2}}\right]_0^k$ **must be replaced by "0" or better**		**A1***
			(5)

Question Number	Scheme/Notes	Marks
8(a)	$I_n = \int_0^k x^n (k-x)^{\frac{1}{2}}\,\mathrm{d}x = \int_0^k x^n (k-x)(k-x)^{-\frac{1}{2}}\,\mathrm{d}x = \int_0^k kx^n (k-x)^{-\frac{1}{2}}\,\mathrm{d}x - \int_0^k x^{n+1}(k-x)^{-\frac{1}{2}}\,\mathrm{d}x$	
Alt	$= \left[-2kx^n (k-x)^{\frac{1}{2}}\right]_0^k + \int_0^k 2knx^{n-1}(k-x)^{\frac{1}{2}}\,\mathrm{d}x + \left[2x^{n+1}(k-x)^{\frac{1}{2}}\right]_0^k - \int_0^k 2(n+1)x^n (k-x)^{\frac{1}{2}}\,\mathrm{d}x$	
Split first	$\Rightarrow 0 + 2knI_{n-1} + 0 - 2(n+1)I_n \Rightarrow (3+2n)I_n = 2knI_{n-1} \Rightarrow I_n = \dfrac{2kn}{3+2n} I_{n-1}$ *	
	For attempts like this award the first 2 method marks **together** for applying the split, expanding **and** applying parts to achieve a correct form. The first accuracy mark can be awarded for a correct expression (limits not required on either part and 'dx's may be missing). As main scheme for the following two marks (note that in this case the first and third terms must both be replaced by "0" or better). There is no mark for just applying the split.	**(5)**

Question Number	Scheme	Notes	Marks
8(b)	$\int_0^k x^2(k-x)^{\frac{1}{2}}\,\mathrm{d}x = \frac{9\sqrt{3}}{280}$ $\qquad I_n = \frac{2kn}{3+2n}I_{n-1}$		
	$I_2 = \frac{4k}{7}I_1 = \frac{4k}{7}\left(\frac{2k}{5}I_0\right)$ or $I_2 = \frac{4k}{7}I_1,\quad I_1 = \frac{2k}{5}I_0$	Attempts I_2 in terms of I_0 **or** I_2 in terms of I_1 **and** I_1 in terms of I_0 Accept with their I_0 substituted if I_0 attempted first. Allow $I_0 = 1$ to be used (i.e., I_0 lost) See note below if only see I_2 in terms of I_1	**M1**
	$I_0 = \int_0^k (k-x)^{\frac{1}{2}}\,\mathrm{d}x = \left[-\frac{2}{3}(k-x)^{\frac{3}{2}}\right]_0^k$	$I_0 = ...(k-x)^{\frac{3}{2}}$ Limits do not have to be seen or applied	**M1**
	$I_2 = \frac{8k^2}{35}\times\frac{2}{3}k^{\frac{3}{2}} \Rightarrow \frac{16}{105}k^{\frac{7}{2}} = \frac{9\sqrt{3}}{280} \Rightarrow k = ...$ Solves an equation of the form $\frac{a}{b}k^{\frac{c}{2}} = \frac{9\sqrt{3}}{280}$ where $a,\ b \in \mathbb{Z}^+$, $\frac{a}{b} \notin \mathbb{Z}$, $c = 5$ or 7 and where the LHS is their I_2. No processing or working requirements just look for a <u>value or numerical expression</u> for *k* from an appropriate equation. May see $k = \mathrm{e}^{\frac{2}{7}\ln\left(\frac{27\sqrt{3}}{128}\right)}$ or other logarithmic work. **Requires both previous M marks**. Note that $\frac{16}{105}k^{\frac{5}{2}} = \frac{9\sqrt{3}}{280} \Rightarrow k = \sqrt[5]{\frac{2187}{16384}}$ or 0.668...		**ddM1**
	$k^{\frac{7}{2}} = \frac{27\sqrt{3}}{128} \Rightarrow k^7 = \frac{2187}{16384} \Rightarrow k = \frac{3}{4}$	Correct exact <u>value</u> for *k* <u>from a correct equation</u>. Not $\sqrt[7]{\frac{2187}{16384}}$ nor $\pm\frac{3}{4}$	**A1**
	Note that if I_2 is only found in terms of I_1 then award the first two marks together when a correct form for I_1 is achieved i.e., $...x(k-x)^{\frac{3}{2}} + ...(k-x)^{\frac{5}{2}}$ or $...(...x + ...k)(k-x)^{\frac{3}{2}}$ Using parts: $I_1 = \left[-\frac{2}{3}x(k-x)^{\frac{3}{2}} - \frac{4}{15}(k-x)^{\frac{5}{2}}\right]_0^k = \frac{4}{15}k^{\frac{5}{2}}$ Using substitution: $u = k - x \Rightarrow I_1 = \int_0^k x(k-x)^{\frac{1}{2}}\,\mathrm{d}x = \left[-\frac{2}{15}(3x+2k)(k-x)^{\frac{3}{2}}\right]_0^k = \frac{4}{15}k^{\frac{5}{2}}$ There are no marks if the reduction formula is not used including direct attempts at I_2 or if $k = \frac{3}{4}$ is arrived at by purely solving the integral equation on a calculator		
			(4)
			Total 9