Question Number	Scheme	Notes	Marks
9	May use **i**, **j**, **k** notation		
9(a)	$\mathbf{n}=\begin{pmatrix}3\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\-2\\2\end{pmatrix}=...\quad\left\{\begin{pmatrix}2\\-5\\-6\end{pmatrix}\right\}$	Calculates the vector product of two vectors in Π_1 (two components correct)	**M1**
	$\begin{pmatrix}5\\3\\0\end{pmatrix}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}=...\quad\{-5\}$	Calculates the scalar product of a point in the plane and their normal. Not dependent but must follow an attempt at a vector product which could be poor, e.g., $3\mathbf{i}+2\mathbf{k}$. Value must be correct if there is no indication of a correct method to evaluate the scalar product.	**M1**
	$\mathbf{r}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}=\begin{pmatrix}5\\3\\0\end{pmatrix}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}\Rightarrow 2x-5y-6z=-5$	Any correct Cartesian equation, e.g., $-2x+5y+6z=5$ $2x-5y-6z+5=0$	**A1**
			(3)
Alt **Sim eqns**	$x=5+3s+t$ $y=3-2t\quad\Rightarrow$ e.g., $y+z=3+s$ $z=s+2t$	Forms simultaneous equations in x, y, z, s and t and obtains an equation that eliminates at least one of s and t	**M1**
	$x=5+3(y+z-3)+\frac{1}{2}z-\frac{1}{2}(y+z-3)$ $x=\frac{5}{2}y+3z-\frac{5}{2}$	M1: Proceeds to an equation in x, y and z only A1: Any correct equation with terms collected	**M1** **A1**
			(3)

Question Number	Scheme	Notes	Marks
9(b) **Way 1**	$2x-5y-6z=-5, \quad 5x-2y+3z=1$ $\Rightarrow$ e.g., $12x-9y=-3$	Uses both plane equations to eliminate one variable. May see $21y+36z=27,\ 21x+27z=15$	**M1**
	e.g., $4x-3y=-1 \Rightarrow \underline{\underline{x=\dfrac{3y-1}{4}}} \Rightarrow \underline{y=\dfrac{4x+1}{3}}$ $3z=1-\dfrac{5(3y-1)}{4}+2y=\dfrac{4-15y+5+8y}{4} \Rightarrow \underline{\underline{z=\dfrac{9-7y}{12}}} \Rightarrow \underline{y=\dfrac{12z-9}{-7}}$ Expresses one variable in terms of the other two (single underlining) or expresses two variables in terms of the other one (double underlining). This work may be seen by setting a variable equal to a parameter to find the other variables in terms of the parameter (or the parameter in terms of the other two variables) e.g., $y=\lambda, \quad x=\mathrm{f}(\lambda), \quad z=\mathrm{g}(\lambda) \quad \left\{\Rightarrow x=\dfrac{-1+3\lambda}{4},\ y=\lambda,\ z=\dfrac{9-7\lambda}{12}\right\}$ $y=\lambda, \quad \lambda=\mathrm{f}(x), \quad \lambda=\mathrm{g}(z) \quad \left\{\Rightarrow \lambda=\dfrac{4x+1}{3},\ y=\lambda,\ \lambda=\dfrac{12z-9}{-7}\right\}$ See examples below. **Requires previous M mark.**		**dM1**
	e.g., $\dfrac{4x+1}{3}=y=\dfrac{12z-9}{-7} \Rightarrow \dfrac{x+\frac{1}{4}}{\frac{3}{4}}=\dfrac{y-0}{1}=\dfrac{z-\frac{3}{4}}{-\frac{7}{12}}$ or e.g., $x=\dfrac{-1+3\lambda}{4},\ y=\lambda,\ z=\dfrac{9-7\lambda}{12} \Rightarrow$ $\Rightarrow \mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}\frac{3}{4}\\1\\-\frac{7}{12}\end{pmatrix}$ or e.g. $\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}9\\12\\-7\end{pmatrix}$	**dd**M1: Correct method to form RHS of vector equation. Allow slips but must not be a clearly incorrect method (e.g., point and direction confused, all non-zero point coordinates the wrong sign, no attempt seen or implied to obtain single coefficients for the variables in the numerator where necessary). Allow this mark if the point is later changed by multiplication e.g., $\left(-\frac{1}{4},\ 0,\ \frac{3}{4}\right)$ becomes $(-1,\ 0,\ 3)$ Condone missing **r** =... Allow this mark if $(\mathbf{r}-\mathbf{a})\times\mathbf{b}(=0)$ or $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ are appropriately used. **Requires both previous M marks.** A1: Any correct **equation** (with any parameter). Do not condone e.g., l = ... Do not isw if the point is changed by multiplication.	**ddM1** **A1**
examples	$x=\dfrac{3y-1}{4}=\dfrac{5-9z}{7} \Rightarrow \dfrac{x-0}{1}=\dfrac{y-\frac{1}{3}}{\frac{4}{3}}=\dfrac{z-\frac{5}{9}}{-\frac{7}{9}}$ or $x=\lambda,\ y=\dfrac{4\lambda+1}{3},\ z=\dfrac{5-7\lambda}{9} \Rightarrow \mathbf{r}=\begin{pmatrix}0\\\frac{1}{3}\\\frac{5}{9}\end{pmatrix}+\lambda\begin{pmatrix}1\\\frac{4}{3}\\-\frac{7}{9}\end{pmatrix}$ $\dfrac{5-7x}{9}=\dfrac{9-7y}{12}=z \Rightarrow \dfrac{x-\frac{5}{7}}{-\frac{9}{7}}=\dfrac{y-\frac{9}{7}}{-\frac{12}{7}}=\dfrac{z-0}{1}$ or $x=\dfrac{5-9\lambda}{7},\ y=\dfrac{12z-9}{-7},\ z=\lambda \Rightarrow \mathbf{r}=\begin{pmatrix}\frac{5}{7}\\\frac{9}{7}\\0\end{pmatrix}+\lambda\begin{pmatrix}-\frac{9}{7}\\-\frac{12}{7}\\1\end{pmatrix}$		**(4)**

Question Number	Scheme	Notes	Marks
9(b) **Way 2** **Finds point and takes vector product of normals**	Work may be minimal if they obtain a correct point. But do not accept just sight of an incorrect point without some evidence of an appropriate method to obtain it.		
	$2x-5y-6z=-5,\quad 5x-2y+3z=1$ Let $y=0\Rightarrow 2x-6z=-5,\quad 5x+3z=1$ or $\Rightarrow$ e.g., $12x-9y=-3$	Assigns a value to one variable to obtain two equations in the other variables or eliminates one variable as in Way 1.	**M1**
	$\Rightarrow 12x=-3\Rightarrow x=-\frac{1}{4},\ y=0,\ z=\frac{3}{4}$ May see $\left(0,\ \frac{1}{3},\ \frac{5}{9}\right)$ or $\left(\frac{5}{7},\ \frac{9}{7},\ 0\right)$	Solves or assigns a value to one variable to find values for the other variables. There is no need to check a point that arises from no working provided it is clear that the previous M mark has been scored. **Requires previous M mark.**	**dM1**
	Note that a point could be obtained via substituting the given form of Π_1 into Π_2 and expanding (M1) and then finding values of *s* and *t* that satisfy the equation and then finding a point (dM1)		
	$\begin{pmatrix}2\\-5\\-6\end{pmatrix}\times\begin{pmatrix}5\\-2\\3\end{pmatrix}=\begin{pmatrix}-27\\-36\\21\end{pmatrix}\Rightarrow\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-27\\-36\\21\end{pmatrix}$	Calculates vector product of normals (two components correct) **and** forms RHS of vector equation (allowing for copying slips but must not confuse point and direction). Allow this mark if the point is later changed by multiplication. Condone missing **r** =... Allow this mark if $(\mathbf{r}-\mathbf{a})\times\mathbf{b}(=0)$ or $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ are appropriately used. **Requires both previous M marks.**	**ddM1**
	$\Rightarrow\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-27\\-36\\21\end{pmatrix}$ or e.g., $\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-9\\-12\\7\end{pmatrix}$	Any correct **equation** in this form (with any parameter). Do not condone e.g., *l* = ... Do not isw if the point is changed by multiplication. Correct points will have the form $\left(\frac{3\alpha-1}{4},\ \alpha,\ \frac{9-7\alpha}{12}\right)$	**A1**
			(4)
Way 3 **2 points**	Finding 2 points on the line and **subtract** for direction e.g., Finds $\left(-\frac{1}{4},\ 0,\ \frac{3}{4}\right)$ (M1dM1 as Way 2) Then finds $\left(0,\ \frac{1}{3},\ \frac{5}{9}\right)\Rightarrow$ direction $=\left(\frac{1}{4},\ \frac{1}{3},\ -\frac{7}{36}\right)\Rightarrow$ forms RHS of vector equation (ddM1) Then A1 for a correct equation		
			(4)
	Correct points/positions include: $\begin{pmatrix}0\\\frac{1}{3}\\\frac{5}{9}\end{pmatrix}\quad\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}\quad\begin{pmatrix}\frac{5}{7}\\\frac{9}{7}\\0\end{pmatrix}\quad\begin{pmatrix}1\\\frac{5}{3}\\-\frac{2}{9}\end{pmatrix}\quad\begin{pmatrix}\frac{1}{2}\\1\\\frac{1}{6}\end{pmatrix}\quad\begin{pmatrix}-\frac{4}{7}\\-\frac{3}{7}\\1\end{pmatrix}\quad\begin{pmatrix}-1\\-1\\\frac{4}{3}\end{pmatrix}\quad\begin{pmatrix}2\\3\\-1\end{pmatrix}$		

Question Number	Scheme	Notes	Marks
9(c)	Note that use of their line from part (b) must be seen to score any marks in (c)		
	$\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}9\\12\\-7\end{pmatrix}=\begin{pmatrix}-\frac{1}{4}+9\lambda\\12\lambda\\\frac{3}{4}-7\lambda\end{pmatrix}$ $4\left(-\frac{1}{4}+9\lambda\right)-3(12\lambda)-\left(\frac{3}{4}-7\lambda\right)=0\Rightarrow 7\lambda=\frac{7}{4}\Rightarrow\lambda=\frac{1}{4}$	Substitutes the parametric form of their line (allow slips but must not clearly confuse position and direction) from (b) into Π_3 and solves for λ The "=0" could be implied by a solution.	**M1**
	$\Rightarrow\left(9\left(\frac{1}{4}\right)-\frac{1}{4},\ 12\left(\frac{1}{4}\right),\ -7\left(\frac{1}{4}\right)+\frac{3}{4}\right)=...$ Substitutes their λ into their line and obtains a point/position vector with values for all coordinates/components. If there is no working at least two coordinates/components should be consistent with their equation or parametric form. Isw if the point/position is altered by multiplication. **Requires previous M mark.**		**dM1**
	$(2,\ 3,\ -1)$	Correct point. No others. Allow $x=...,\ y=...,\ z=...$ and condone as a position vector. Do not isw.	**A1**
			(3)
			Total 10
			PAPER TOTAL 75

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