Question	Scheme	Marks
1	$2\sinh^2 x+3\cosh x=7 \Rightarrow 2(\cosh^2 x-1)+3\cosh x=7$	M1
	$\Rightarrow 2\cosh^2 x+3\cosh x-9=0$	A1
	$\Rightarrow (2\cosh x-3)(\cosh x+3)=0 \Rightarrow \cosh x=\frac{3}{2}, \cancel{-3}$	M1
	$\Rightarrow x=\text{arcosh}\frac{3}{2}=(\pm)\ln\left(\frac{3}{2}+\sqrt{\frac{9}{4}-1}\right)$	M1
	$x=\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right),\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$	A1A1
		(6)
Alt	$2\sinh^2 x+3\cosh x=7 \Rightarrow 2\left(\frac{e^x-e^{-x}}{2}\right)^2+3\left(\frac{e^x+e^{-x}}{2}\right)=7$	M1
	$\Rightarrow e^{4x}+3e^{3x}-16e^{2x}+3e^x+1=0$	A1
	$\Rightarrow (e^{2x}+6e^x+1)(e^{2x}-3e^x+1)=0 \Rightarrow e^x=...$	M1
	$e^x=\cancel{-3\pm 2\sqrt{2}}, \frac{3}{2}\pm\frac{\sqrt{5}}{2} \Rightarrow x=\ln\left(\frac{3}{2}\pm\frac{\sqrt{5}}{2}\right)$	M1
	$x=\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right),\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$	A1A1
		(6)
		(6 marks)

Notes:

M1: Attempts to use the correct identity $\cosh^2 x-\sinh^2 x=1$ to form an equation in cosh *x* only. (Allow slips rearranging a correctly stated identity.)

A1: Correct 3 term quadratic in cosh *x*. The "=0" may be implied by their attempt to solve.

M1: Solves the 3 term quadratic (usual rules) leading to at least one exact real value for cosh *x* (which may be negative). The negative value may not be seen, but need not be rejected at this stage. If using a calculator they must state at least one correct root for their equation.

M1: Forms a solution in terms of natural logarithms from a positive solution to their equation using the correct formula or via using the correct exponential form and solving a 3 term quadratic in e^x.

A1: One value correct.

A1: Both values correct, accepting equivalents, and no others. Equivalents are e.g.

$x=\pm\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$, $\pm\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$ Condone missing brackets.

Alt

M1: Substitutes **correct** definitions for sinh *x* and cosh *x* in terms of exponentials.

A1: Correct simplified quartic in e^x.

M1: Solves their 5 term quartic as far as e^x = …. to achieve exact values (at least one). There should be a recognisable attempt at solving, e.g. via a product of two quadratics, with at least first and last terms

matching their quartic, but allow answers only from a calculator – but must be correct and exact for their equation.

M1: Proceeds to a solution in terms of natural logarithms from a positive solution to their equation using the correct formula. Not dependent, so this may be scored from decimal solutions to the quartic, for proceeding from $e^x = \ldots$ to $x = \ln\ldots$

A1: One value correct.

A1: Both values correct, accepting equivalents, and no others.

There may be other variations via substitutions etc.

M1: Attempts to use the correct identities to form an equation in one "variable".

A1: Correct simplified equation.

M1: Full attempt to solve the equation to at least $\cosh x = \ldots$ or $e^x = \ldots$ (exact)

M1: Forms a solution in terms of natural logarithms from a valid solution to their equation using the correct formula.

A1: One value correct.

A1: Both values correct, accepting equivalents, and no others. Equivalents are e.g.

$x = \pm\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$, $\pm\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$ Condone missing brackets.