Question	Scheme	Marks
3(i)	$\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x$	
	$\left\{4x^2+12x+25=4\left(x^2+3x\right)+25\right\}$ $=4\left(x+\frac{3}{2}\right)^2+\ldots$ or $4x^2+12x+25=\left(2x+3\right)^2+\ldots$	M1
	$4\left(x+\frac{3}{2}\right)^2+16$ or $\left(2x+3\right)^2+16$	A1
	$\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x=\frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+4}\,\mathrm{d}x=k\arctan\left(\frac{"x+\frac{3}{2}"}{"2"}\right)(+c)$ or $\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x=\int \frac{1}{\left(2x+3\right)^2+4^2}\,\mathrm{d}x=k\arctan\left(\frac{"2x+3"}{"4"}\right)(+c)$	M1
	$=\frac{1}{8}\arctan\left(\frac{2x+3}{4}\right)(+c)$ (oe)	A1
		(4)
(ii)	$\int_3^9 \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x=\ln a$	
	$x^2+4x-17=\left(x+2\right)^2-4-17$	B1
	$\int \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x=\int \frac{1}{\sqrt{(x+2)^2-21}}\,\mathrm{d}x=\operatorname{arcosh}\left(\frac{x+2}{\sqrt{21}}\right)$ or $\ln\left(x+2+\sqrt{(x+2)^2-21}\right)$ or $\ln\left(\frac{x+2}{\sqrt{21}}+\sqrt{\left(\frac{x+2}{\sqrt{21}}\right)^2-1}\right)$ Alt: $u=x+2\Rightarrow\int \frac{1}{\sqrt{u^2-21}}\,\mathrm{d}u=\operatorname{arcosh}\left(\frac{u}{\sqrt{21}}\right)$ or $\ln\left(u+\sqrt{u^2-21}\right)$ etc	M1 A1
	$\int_3^9 \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x=\operatorname{arcosh}\left(\frac{11}{\sqrt{21}}\right)-\operatorname{arcosh}\left(\frac{5}{\sqrt{21}}\right)$ or $\ln\left(11+\sqrt{11^2-21}\right)-\ln\left(5+\sqrt{5^2-21}\right)$	M1
	$\left(=\ln\left(\frac{11}{\sqrt{21}}+\sqrt{\frac{121}{21}-1}\right)-\ln\left(\frac{5}{\sqrt{21}}+\sqrt{\frac{25}{21}-1}\right)\right)=\ln\left(\frac{11+\sqrt{100}}{5+\sqrt{4}}\right)$	dM1
	$=\ln 3$	A1
		(6)
		(10 marks)
Notes:		

(i)

M1: Factors out the 4 and completes the square achieving $4\left(x+\frac{3}{2}\right)^2+\ldots$ or without factoring the 4 achieves $(2x+3)^2+\ldots$ where … is anything other than 25. Condone for this mark $4\int\frac{1}{\left(x+\frac{3}{2}\right)^2+\ldots}\,\mathrm{d}x$ but this is A0 if no independent correct expression is shown.

A1: Correct completed square form (constant must be gathered).

M1: Attempts the integration with arctan used. Look for $k\arctan\left(l\times\text{their linear term}\right)$. Allow with any *k* and *l*, including 1 or $\frac{1}{4}$. (May be via a substitution but must return to integral in *x* of the correct form for the mark.)

If an incorrect completion of the square results in $(ax+b)^2-c^2$ then this mark may be awarded for $\ldots\ln\left(\frac{ax+b-c}{ax+b+c}\right)$

A1: Correct answer, need not be fully simplified but must have at least combined multiple fractions, e.g. $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ must have been combined. Accept alternative forms of arctan, e.g. $\tan^{-1}$ or atan.

(ii)

B1: Correct completed square form for the quadratic (need not be fully simplified).

M1: Attempts the integration to either arcosh or ln form. Form should be correct for their $x+2$ and *a*. Allow with ln| | and give benefit of doubt for the M if mod is required. They may use a substitution for this mark, $u=x+2$, and may be scored for the integral in terms of *u*. If a substitution such as $x+2=\cosh(u)$ or $x+2=\sqrt{21}\sec u$ is used they must carry out the full process – with supporting working shown - to reach an integral in *u* **with correct limits identified** as part of the method, or reversal of the substitution must carried out.

A1: Correct integral (may be in terms of *u* if the substitution $u=x+2$ is used), or correct integral **and** correct limits if a hyperbolic substitution is used. Ignore spurious statements of equality if they give both forms, and award for either correct form. Allow with moduli in the logs.

Note that $x+2=\sqrt{21}\sec u$ will lead to an integral $\left[\ln\left|\tan u+\sec u\right|\right]_{\operatorname{arcsec}\frac{5}{\sqrt{21}}}^{\operatorname{arcsec}\frac{11}{\sqrt{21}}}$

M1: Applies the limits 3 and 9 to their integral of a correct form (allowing a minor slip, e.g. a sign error inside a ln), or correct limits for their variable if they use a substitution, in an integral of the correct form (may be scored if M0A0 was awarded for insufficient evidence). Evidence of the application of limits must be seen – do not accept solutions which go directly to the answer. All steps of working are asked for in the question.

dM1: Simplifies to a single ln term, applying the arcosh ln (allowing a minor slip, e.g. a sign error inside a ln) formula first if necessary. Again, there must be sufficient evidence after substitution before the final simplified answer.

A1: cso