Question	Scheme	Marks
4(a)	$\begin{pmatrix}4&0&2\\0&4&a\\2&a&20/3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}10\\...\\...\end{pmatrix}$	**M1**
	$\lambda_1=10$	**A1**
		(2)
(b)	$\begin{pmatrix}4&0&2\\0&4&a\\2&a&20/3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}10\\20\\30\end{pmatrix}\Rightarrow 8+3a=20 \text{ or } 2+2a+\frac{20}{3}\times 3=30$	**M1**
	$\Rightarrow a=4$*	**A1***
		(2)
(c)	$\begin{vmatrix}4-\lambda&0&2\\0&4-\lambda&4\\2&4&\frac{20}{3}-\lambda\end{vmatrix}=(4-\lambda)\left((4-\lambda)\left(\frac{20}{3}-\lambda\right)-16\right)+2\left(0-2(4-\lambda)\right)$	**M1**
	$\left[-\lambda^3+\frac{44}{3}\lambda^2-\frac{148}{3}\lambda+\frac{80}{3}=0\right]\Rightarrow\frac{1}{3}(4-\lambda)(3\lambda^2-32\lambda+20)=0\Rightarrow\lambda=...,...$	**dM1**
	$\lambda=4,\frac{2}{3}(,10)$	**A1**
		(3)
(d)	$\begin{pmatrix}4&0&2\\0&4&4\\2&4&20/3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\lambda\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow\begin{cases}(4-\lambda)x+2z=0\\(4-\lambda)y+4z=0\\2x+4y+\left(\frac{20}{3}-\lambda\right)z=0\end{cases}\Rightarrow ...$	**M1**
	$\lambda=\frac{2}{3}\Rightarrow k\begin{pmatrix}3\\6\\-5\end{pmatrix}$ or $\lambda=4\Rightarrow m\begin{pmatrix}2\\-1\\0\end{pmatrix}$	**A1**
	Normalised $\Rightarrow\frac{v}{\|v\|}=...\ \lambda=\frac{2}{3}\Rightarrow\frac{1}{\sqrt{70}}\begin{pmatrix}3\\6\\-5\end{pmatrix}$ and $\lambda=4\Rightarrow\frac{1}{\sqrt{5}}\begin{pmatrix}2\\-1\\0\end{pmatrix}$; $\lambda=10\Rightarrow\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix}$	**M1** **A1;B1**
		(5)
(e)	$\mathbf{P}=\begin{pmatrix}1/\sqrt{14}&2/\sqrt{5}&3/\sqrt{70}\\2/\sqrt{14}&-1/\sqrt{5}&6/\sqrt{70}\\3/\sqrt{14}&0&-5/\sqrt{70}\end{pmatrix}$, $\mathbf{D}=\begin{pmatrix}10&0&0\\0&4&0\\0&0&2/3\end{pmatrix}$	**B1ft** **B1ft**
		(2)
		(14 marks)

Notes:

(a)

M1: Correct attempt to find the eigenvalue. Need only see top row calculated. May come from attempts at using $\left(\mathbf{M}-\lambda\mathbf{I}\right)\mathbf{v}=\mathbf{0}$

A1: Correct value.

(b)

M1: Sets up a correct equation in *a* for their eigenvalue in (a). Must be extracted from the matrix equation. May come from attempts at using $\left(\mathbf{M}-\lambda\mathbf{I}\right)\mathbf{v}=\mathbf{0}$. Condone any unused incorrect equations.

A1*: Shows that $a = 4$ from correct work.

(c)

M1: Attempts to expand $\det(\mathbf{M}-\lambda\mathbf{I})$. This may be along any row or down any column, or via a "shoelace" approach. Condone sign slips, but the overall structure should be correct. Working must be shown – do not accept solutions that just state the factorised form. The expansion must be seen in some form. Accept alternative approaches.

dM1: Sets the characteristic polynomial equal to zero (which may be implied) and solves by a valid method (may be by calculator) – dependent, so as a minimum, an attempt at the expansion must have been seen but the cubic need not have been fully simplified.

A1: Correct remaining eigenvalues.

(d) Note – you may allow recovery of the final 3 marks in this part if the relevant normalisation work is completed in part (e).

M1: Sets up the eigenvector equations in general terms or for one of their eigenvalues and makes some attempt to use them (allow even for the given eigenvector for this mark). Working must be shown, M0 if correct vectors are stated with no supporting working.

A1: Either of the two remaining eigenvectors found, need not be normalised for this mark. Must be given as a vector, but accept row vectors rather than column vectors.

M1: Applies a correct process to normalise at least one of their vectors. This may be done on the given eigenvector (working need not be shown for this one).

A1: Both normalised eigenvectors for $\lambda = 4, \frac{2}{3}$ correct. (Row or column vectors acceptable.)

B1: For correct normalised vector for eigenvalue 10. (Row or column vector acceptable.)

You may see $\lambda=\frac{2}{3}\Rightarrow\begin{pmatrix}3\sqrt{70}/70\\3\sqrt{70}/35\\-\sqrt{70}/14\end{pmatrix}$ and $\lambda=4\Rightarrow\begin{pmatrix}2\sqrt{5}/5\\-\sqrt{5}/5\\0\end{pmatrix}$; $\lambda=10\Rightarrow\begin{pmatrix}\sqrt{14}/14\\\sqrt{14}/7\\3\sqrt{14}/14\end{pmatrix}$ or other equivalent versions of these vectors, e.g $\lambda=\frac{2}{3}\Rightarrow\frac{\sqrt{70}}{14}\begin{pmatrix}3/5\\6/5\\-1\end{pmatrix}$

(e)

B1ft: For either a correct **P** for their **normalised** eigenvectors providing they are non-zero **or** for a correct **D** for their eigenvalues.

B1ft: For both a correct matching **P** and **D** for their **normalised** eigenvectors and their eigenvalues, providing there is no zero column or row.

Note: If **D** and **P** are not labelled then treat their diagonal matrix as **D** and other matrix as **P**. If they are incorrectly labelled (ie **D** as **P**) then withhold the second B (if earned).