4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & \frac{20}{3} \end{pmatrix}$$

where a is a constant.

Given that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of **M**,

(a) determine the corresponding eigenvalue for this eigenvector.

(2)

(b) Hence show that $a = 4$

(2)

Determine

(c) the other two eigenvalues of **M**,

(3)

(d) a normalised eigenvector for each eigenvalue of **M**.

(5)

(e) Hence determine a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{MP} = \mathbf{D}$$

(2)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 14 marks)