Question	Scheme	Marks
7(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{b\cos\theta}{-a\sin\theta}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2x}{a^2}\times\frac{b^2}{2y}$	**M1**
	$m_N=\frac{a\sin\theta}{b\cos\theta}$	**A1**
	Equation is $y-b\sin\theta=\frac{a\sin\theta}{b\cos\theta}(x-a\cos\theta)$	**M1**
	$\Rightarrow by-b^2\sin\theta=ax\tan\theta-a^2\sin\theta\Rightarrow by=ax\tan\theta+(b^2-a^2)\sin\theta$ *	**A1***
		(4)
(b)	B is $(0,-b)\Rightarrow -b^2=(b^2-a^2)\sin\theta$	**B1**
	Area $OBP=\frac{3}{4}b^2\Rightarrow\frac{1}{2}b\times a\cos\theta=\frac{3}{4}b^2$	**M1**
	$\Rightarrow 2a\cos\theta=3b\Rightarrow b^2=\frac{4}{9}\cancel{a^2}\cos^2\theta=\left(\cancel{a^2}-\frac{4}{9}\cancel{a^2}\cos^2\theta\right)\sin\theta$	**dM1**
	$\Rightarrow 4(1-\sin^2\theta)=(9-4(1-\sin^2\theta))\sin\theta$ $\Rightarrow 4\sin^3\theta+4\sin^2\theta+5\sin\theta-4=0$	**ddM1**
	$\Rightarrow(2\sin\theta-1)(2\sin^2\theta+3\sin\theta+4)=0\Rightarrow\sin\theta=\frac{1}{2}$ *	**A1***
		(5)
(c)	$b=\frac{4}{3}\times\frac{1}{2}a\times\sqrt{1-\left(\frac{1}{2}\right)^2}=\ldots\left(=\frac{\sqrt{3}}{3}a\right)$	**M1**
	P is $\left(\frac{\sqrt{3}}{2}a,\frac{\sqrt{3}}{6}a\right)$	**B1** **A1**
		(3)
		(12 marks)

Notes:

(a)

M1: A correct method for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of θ (from parametric differentiation) or in terms of x and y (from implicit or Cartesian approaches).

A1: Correct gradient for the normal in terms of θ

M1: Correct method for the normal through P applied. There must have been a correct attempt at the gradient of the normal (ie negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$, or directly finding $-\frac{\mathrm{d}x}{\mathrm{d}y}$). Allow recovery from gradients in terms of x and y only if correct substitution for them occurs. If using $y = mx + c$ they must proceed as far as finding c.

A1*: Correct completion to the equation given with no errors seen in working and an intermediate step (e.g. if using $y = mx + c$ they must substitute c back in before proceeding to the given answer).

(b)

B1: States or identifies the point B is $(0,-b)$ and substitutes into the normal equation to obtain the correct equation shown or equivalent.

M1: Correct attempt at the area and sets equal to $\frac{3}{4}b^2$ to form a second equation. Note there are other possible methods for the area of the triangle e.g. "shoelace"/determinant method, or use of separate triangles. Some will essentially give the same result as in the scheme, but others will give more complicated formulae, making the next M much more challenging (and unlikely to be gained).

dM1: Solves the equations simultaneously to achieve an equation in θ only. Alternatively, eliminates $\sin\theta$ and forms a polynomial equation in a^2 and b^2. This latter is unlikely to make further progress.

ddM1: Rearranges to an expanded (but not necessarily fully simplified) cubic in $\sin\theta$. For equivalent working in a^2 or b^2 they must proceed as far as finding a relationship between a^2 and b^2.

A1*: Fully correct work, showing the factorised cubic to justify the given value of sin θ. No need to show there is only one solution (we allow the factorised to imply the quadratic factor does not reduce). Note substitution of $\sin\theta=\frac{1}{2}$ into the equation is A0 as it provides no evidence of no other solutions.

For working in a and b the factorisation to deduce the relationship between a^2 and b^2 must have been seen.

(c)

M1: Correct process to find b in terms of a

B1: x coordinate correct

A1: y coordinate correct, accept $\frac{a}{2\sqrt{3}}$.

(b) Working in a and b:

$$\sin^2\theta+\cos^2\theta=1\Rightarrow\frac{b^4}{\left(b^2-a^2\right)^2}+\frac{9b^2}{4a^2}=1\Rightarrow 4a^2b^4+9b^2\left(b^2-a^2\right)^2=4a^2\left(b^2-a^2\right)^2 \quad \text{dM1}$$

$$\Rightarrow 9b^6-18a^2b^4+17a^4b^2-4a^6=0\Rightarrow\left(3b^2-a^2\right)\left(3b^4-5a^2b^2+4a^4\right)=0$$

$$\Rightarrow a^2=3b^2 \quad \text{ddM1}$$

$$\Rightarrow\sin\theta=-\frac{b^2}{b^2-3b^2}=-\frac{1}{-2}=\frac{1}{2} \quad \text{A1}$$

Note: A lot of solutions are not using that the coordinate of B is $(0,-b)$ but instead use $\sin\theta=\frac{1}{2}$ to show $\sin\theta=\frac{1}{2}$. These will score at most B0M1M0M0A0. as they have no equations to solve simultaneously.