Question	Scheme	Marks
8	$y = e^{3x}\cosh 2x$ (See notes for Alt via $n = 0$ for M1A1) $\Rightarrow \frac{dy}{dx} = 3e^{3x}\cosh 2x + e^{3x} \times 2\sinh 2x$ or $e^{3x}\left(3\cosh 2x + 2\sinh 2x\right)$	**M1**
	$= e^{3x}\left(\frac{5+1}{2}\cosh 2x + \frac{5-1}{2}\sinh 2x\right)$ (so the result is true for $n = 1$)	**A1**
	(Assume the result is true for $n = k$, then) $\frac{d^{k+1}y}{dx^{k+1}} = 3e^{3x}\left(\frac{5^k+1}{2}\cosh 2x + \frac{5^k-1}{2}\sinh 2x\right) + e^{3x}\left(\left(5^k+1\right)\sinh 2x + \left(5^k-1\right)\cosh 2x\right)$	**M1**
	$= \left(3\times\frac{5^k+1}{2} + 5^k - 1\right)e^{3x}\cosh 2x + \left(3\times\frac{5^k-1}{2} + 5^k + 1\right)e^{3x}\sinh 2x$ $= e^{3x}\left(\frac{5\times 5^k+1}{2}\cosh 2x + \frac{5\times 5^k-1}{2}\sinh 2x\right)$	**dM1**
	$= e^{3x}\left(\frac{5^{k+1}+1}{2}\cosh 2x + \frac{5^{k+1}-1}{2}\sinh 2x\right)$	**A1**
	Hence the result is also true for $n = k+1$, so <u>if true for $n = k$ then true for $n = k + 1$</u>, and as also <u>true for $n = 1$</u>, so the result is <u>true for all positive integers</u>.	**A1**
		(6)
		(6 marks)

Notes:

Note: Condone use of n instead of k throughout the inductive step. If reasoned correctly full marks are available. Also note that going from $n = k - 1$ to $n = k$ is a correct method.

Note: Apply benefit of the doubt if poor handwriting makes it difficult to distinguish between 5^k and $5k$ in some places (ie powers written too lower) if they are treated correctly in subsequent steps. This will not be counted as errors affecting the A's.

M1: Attempts the first derivative - form must be correct, but coefficients may be incorrect. Alternatively, allow for substitution of $n = 0$ into the right hand side – accept as minimum seeing $e^{3x}\left(\frac{1+1}{2}\cosh 2x + \frac{1-1}{2}\sinh 2x\right)$.

A1: Correct derivative and reaches appropriate form, either by factoring out the e^{3x} or expanding the RHS of result for $n = 1$ and comparing, to be able to deduce the result is true for $n = 1$ (which may be tacit for this mark). If expanding the RHS the substitution of the $n = 1$ should be clear.

Either way a minimum of $e^{3x}\left(\frac{5+1}{2}\cosh 2x + \frac{5-1}{2}\sinh 2x\right)$ must be seen in their working in order to award this mark.

Alternatively, substituting $n = 0$, proceeds from the minimum shown to reach $e^{3x}\cosh 2x = y$ to establish true for $n = 0$.

M1: (Makes the inductive assumption and) attempts the $(k+1)$-th derivative from the k-th derivative (but see first note above). Allow slips in coefficients.

dM1: Gathers the sinh 2x and cosh 2x terms and factors out these terms to unsimplified coefficients. The e^{3x} may or may not be factored out, or missing, or incorrect at this stage. Accept either form shown (or equivalent unsimplified coefficients). There must be a step with unsimplified gathered coefficients between the differentiation and final line shown in the scheme to score this mark.

A1: Reaches the correct form from correct work. The e^{3x} must be factored out, or expansion of the RHS to demonstrate equivalents seen. Must have the "k+1" showing. Depends on the previous two method marks.

A1: Makes appropriate concluding sentence covering the points indicated in scheme at some stage in the proof. Depends on all method marks having been scored and all work being correct. Must have reached at least the second line shown in the dM mark, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown).

Note M1A0M1dM1A0A1 is possible if the previous A marks have only been withheld due to the lack of sufficient detail shown but the working and intent is clear (e.g only reaching the final line of the dM box).

8 Alt via exponentials	$y = e^{3x}\cosh 2x = \frac{e^{5x}+e^{x}}{2} \quad \Rightarrow \frac{dy}{dx} = \frac{5e^{5x}+e^{x}}{2}$	**M1**
	$= e^{3x}\left(\frac{5e^{2x}+e^{-2x}}{2}\right) = e^{3x}\left(\frac{(5+1)\left(e^{2x}+e^{-2x}\right)+(5-1)\left(e^{2x}-e^{-2x}\right)}{4}\right)$ $= e^{3x}\left(\frac{5+1}{2}\cosh 2x + \frac{5-1}{2}\sinh 2x\right)$ so the result is true for n = 1	**A1**
	(Assume the result is true for $n = k$, so $\frac{d^k y}{dx^k} = e^{3x}\left(\frac{5^k+1}{2}\frac{e^{2x}+e^{-2x}}{2} + \frac{5^k-1}{2}\frac{e^{2x}-e^{-2x}}{2}\right) = \frac{5^k}{2}e^{5x} + \frac{1}{2}e^{x}$ then) $\frac{d^{k+1}y}{dx^{k+1}} = \frac{5^k}{2}\times 5e^{5x} + \frac{1}{2}e^{x}$	**M1**
	$= \frac{5^{k+1}}{2}e^{5x} + \frac{1}{2}e^{x} = \frac{e^{3x}}{4}\left(2\times 5^{k+1}e^{2x} + 2e^{-2x}\right)$ $= \frac{e^{3x}}{4}\left(5^{k+1}e^{2x} + 5^{k+1}e^{-2x} + 5^{k+1}e^{2x} - 5^{k+1}e^{-2x} + e^{2x} + e^{-2x} + e^{-2x} - e^{2x}\right)$ $= \frac{e^{3x}}{2}\left(5^{k+1}\cosh 2x + 5^{k+1}\sinh 2x + \cosh 2x - \sinh 2x\right)$	**dM1**
	$= e^{3x}\left(\frac{5^{k+1}+1}{2}\cosh 2x + \frac{5^{k+1}-1}{2}\sinh 2x\right)$	**A1**
	Hence the result is also true for $n = k+1$, so <u>if true for $n = k$ then true for $n = k+1$</u>, and as also <u>true for n = 1</u>, so the result is <u>true for all positive integers</u>.	**A1**
		(6)
		(6 marks)

Notes:

M1: Replaces by the correct exponential forms and differentiates - form must be correct, but coefficients may be incorrect.

A1: Correct derivative and reaches appropriate form to deduce the result is true for $n = 1$ (which may be tacit for this mark). If expanding the RHS the substitution of the $n = 1$ should be clear. The same minimum as main scheme is expected except that the cosh $2x$ and sinh $2x$ may have been replaced first.

M1: (Makes the inductive assumption and) attempts the $(k+1)$-th derivative from the k-th derivative. Allow slips in coefficients.

dM1: Attempts to introduce the extra terms to establish the cosh $2x$ and sinh $2x$ in the expression. Allow slips, but the intention must be clear. Alternatively, considers the right hand side for $k + 1$, substitutes exponential forms and expands and simplifies.

A1: Fully correct work to establish the inductive step. The final expression must be reached (with the intermediate dM1 gained) from correct work, or have been used to work back to matching expressions.

A1: Makes appropriate concluding sentence covering the points indicated in scheme at some stage in the proof. Depends on all method marks having been scored and all work being correct. Must have reached at least the final line shown in the dM mark or equivalent work in reverse, and made an attempt at checking $n = 1$ (though the first A mark need not have been scored if insufficient detail shown).

Note M1A0M1dM1A0A1 is possible if the previous A marks have only been withheld due to the lack of sufficient detail shown but the working and intent is clear (e.g only reaching the final line of the dM box).

Note: Some attempts may try to use integration to work backwards from $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}$ to $\frac{\mathrm{d}^{k}y}{\mathrm{d}x^{k}}$. Such methods may use hyperbolics or exponentials and the scheme will apply similarly.

M1A1: as per main scheme.

M1: Considers result for $n = k + 1$ and attempts to integrate twice by parts, achieving correct form.

Let $A=\frac{5^{k+1}+1}{2}\cosh 2x+\frac{5^{k+1}-1}{2}\sinh 2x, \quad B=\left(5^{k+1}+1\right)\sinh 2x+\left(5^{k+1}-1\right)\cosh 2x$

$$I=\int\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}\mathrm{d}x=\frac{1}{3}\mathrm{e}^{3x}\left(A\right)-\frac{1}{3}\int\mathrm{e}^{3x}B\,\mathrm{d}x=\frac{1}{3}\mathrm{e}^{3x}\left(A\right)-\frac{1}{9}\mathrm{e}^{3x}B+\frac{4}{9}\int\mathrm{e}^{3x}A\,\mathrm{d}x$$

Or in exponentials

$$\int\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}\mathrm{d}x=\int\left(\frac{5^{k+1}+1}{2}\frac{\mathrm{e}^{5x}+\mathrm{e}^{x}}{2}+\frac{5^{k+1}-1}{2}\frac{\mathrm{e}^{5x}-\mathrm{e}^{x}}{2}\right)\mathrm{d}x=\frac{5^{k+1}+1}{2}\frac{\frac{1}{5}\mathrm{e}^{5x}+\mathrm{e}^{x}}{2}+\frac{5^{k+1}-1}{2}\frac{\frac{1}{5}\mathrm{e}^{5x}-\mathrm{e}^{x}}{2}$$

dM1: Makes I the subject

$$\Rightarrow 9I=3\mathrm{e}^{3x}\left(A\right)-\mathrm{e}^{3x}B+4I\Rightarrow I=\frac{1}{5}\mathrm{e}^{3x}\left(3A-B\right)$$

Or: $=\frac{\mathrm{e}^{3x}}{5}\left(\frac{5^{k+1}+1}{2}\frac{\mathrm{e}^{2x}+5\mathrm{e}^{-2x}}{2}+\frac{5^{k+1}-1}{2}\frac{\mathrm{e}^{2x}-5\mathrm{e}^{-2x}}{2}\right)=\frac{\mathrm{e}^{3x}}{5}\left(\frac{2\times5^{k+1}\mathrm{e}^{2x}+10\mathrm{e}^{-2x}}{4}\right)$ leading to identifying

$....\left(\text{reverse work of exponentials in main scheme}\right)$

exponential forms for the hyperbolics.

A1: Achieves the correct result and gives due consideration to the constant of integration.

$$\Rightarrow\frac{\mathrm{d}^{k}y}{\mathrm{d}x^{k}}=I=\frac{1}{5}\mathrm{e}^{3x}\left(\frac{5^{k+1}+5}{2}\cosh 2x+\frac{5^{k+1}-5}{2}\sinh 2x\right)=\mathrm{e}^{3x}\left(\frac{5^{k}+1}{2}\cosh 2x+\frac{5^{k}-1}{2}\sinh 2x\right)$$

A1: Per main scheme (except allow if constant of integration is not considered).

If you are unsure in a particular case use the review system.

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