Question Number	Scheme	Marks
In all parts, accept alternative notation for arsinh and arcosh such as $\sinh^{-1}$ and $\cosh^{-1}$ etc		
2(i)	$4x^2+8x+9=4(x+1)^2+5 \quad \text{or} \quad 4x^2+8x+9=(2x+2)^2+5$	B1
	$\int \frac{1}{\sqrt{4x^2+8x+9}}\,\mathrm{d}x=\int \frac{1}{\sqrt{4(x+1)^2+5}}\,\mathrm{d}x=\frac{1}{2}\operatorname{arsinh}\left(\frac{2(x+1)}{\sqrt{5}}\right)(+c)$ o.e.	M1A1
		(3)
(ii)	$\int \operatorname{arcosh}3x\,\mathrm{d}x=x\operatorname{arcosh}3x-\int \frac{3x}{\sqrt{9x^2-1}}\,\mathrm{d}x$	M1 A1
	$=\ldots-\frac{1}{3}(9x^2-1)^{\frac{1}{2}}$ or $\ldots-\frac{1}{3}\sinh(\operatorname{arcosh}3x)$	M1
	$\int \operatorname{arcosh}3x\,\mathrm{d}x=x\operatorname{arcosh}3x-\frac{1}{3}(9x^2-1)^{\frac{1}{2}}(+c)$ or $x\operatorname{arcosh}3x-\left(x^2-\frac{1}{9}\right)^{\frac{1}{2}}(+c)$ or $\int \operatorname{arcosh}3x\,\mathrm{d}x=x\operatorname{arcosh}3x-\frac{1}{3}\sinh(\operatorname{arcosh}3x)(+c)$	A1
		(4)
ALT	$u=\operatorname{ar}\cosh 3x \Rightarrow x=\frac{1}{3}\cosh u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{3}\sinh u$ $\int \operatorname{arcosh}3x\,\mathrm{d}x=\int \frac{1}{3}u\sinh u\,\mathrm{d}u=\frac{u}{3}\cosh u-\int \frac{1}{3}\cosh u\,\mathrm{d}u$	M1A1
	$\frac{u}{3}\cosh u-\frac{1}{3}\sinh u\ (+c)$	M1
	$x\operatorname{ar}\cosh 3x-\frac{1}{3}\sinh(\operatorname{ar}\cosh 3x)(+c)$ or $x\operatorname{arcosh}3x-\frac{1}{3}(9x^2-1)^{\frac{1}{2}}(+c)$ or $x\operatorname{arcosh}3x-\left(x^2-\frac{1}{9}\right)^{\frac{1}{2}}(+c)$	A1
		(4)
		Total 7

Notes:

(i)

B1: Correct completion of the square. May be implied by an integral of the form $\frac{1}{2}\int \frac{1}{\sqrt{(x+1)^2+\frac{5}{4}}}\,\mathrm{d}x$

M1: Obtains $k\,\mathrm{ar}\sinh \mathrm{f}(x)$ $(k \in \mathbb{R})$ or other equivalent form e.g.

$\frac{1}{2}\ln\left[(x+1)+\sqrt{(x+1)^2+\frac{5}{4}}\right]$ or $\frac{1}{2}\ln\left[(2x+2)+\sqrt{4x^2+8x+9}\right]$ etc

A1: Correct integration (condone omission of + *c*). Missing '*h*' is A0 so arsin is A0 but condone arcsinh *x*.

(ii)

M1: Applies integration by parts to obtain an expression of the form $ax\mathrm{arcosh}3x - b\int \frac{x}{\sqrt{cx^2-1}}\,\mathrm{d}x$

A1: Correct expression (condone missing 'd*x*' etc)

M1: Integrates $b\int \frac{x}{\sqrt{\pm(cx^2-1)}} \to k\sqrt{\pm(cx^2-1)}$ or $b\int \frac{x}{\sqrt{cx^2-1}} \to k\sinh(\mathrm{arcosh}3x)$

A1: Fully correct integration (condone omission of + *c*)

ALT

M1: Applies integration by parts (after substitution of $u = \mathrm{ar}\cosh 3x$) to obtain an expression of the form $au\cosh u - \int b\cosh u\,\mathrm{d}u$

A1: Correct expression (condone missing 'du' etc)

M1: Integrates $\int b\cosh u\,\mathrm{d}u \to c\sinh u$

A1: Replaces *x* and obtains a fully correct integration (condone omission of + *c*)

Note: attempts that re-write arcosh3*x* in ln form then attempt to integrate score zero.