Question Number	Scheme	Marks
	Accept vectors in **i, j, k** form or column vector form throughout	
3(a)	$\lambda = (1\times1)+(-1\times2)+(4\times1) = ...$	M1
	$\lambda = 3$	A1
		(2)
(b)	$\begin{pmatrix}1 & -1 & 4\\ 3 & a & b\\ a & 1 & b\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}"\lambda"\\"2\lambda"\\"\lambda"\end{pmatrix} \Rightarrow \begin{matrix}3+2a+b="2\lambda"\\ a+2+b="\lambda"\end{matrix} \Rightarrow a=..., b=...$	M1
	$a=2, b=-1$	A1
		(2)
(c)(i)	$\left\|\mathbf{M}-\lambda\mathbf{I}\right| = \begin{vmatrix}1-\lambda & -1 & 4\\ 3 & 2-\lambda & b\\ a & 1 & -1-\lambda\end{vmatrix}$ $=(1-\lambda)\left[(\lambda-2)(\lambda+1)+1\right]+3(-1-\lambda)+2+4\left[3-2(2-\lambda)\right]$ (via first row) $=(1-\lambda)\left[(\lambda-2)(\lambda+1)+1\right]-3\left[(1+\lambda)-4\right]+2\left[1-4(2-\lambda)\right]$ (via first column) $=(1-\lambda)(2-\lambda)(-1-\lambda)+2+12-8(2-\lambda)+(1-\lambda)+3(-1-\lambda)$ (Sarrus)	M1
	$\lambda^3-2\lambda^2-5\lambda+6=0 \Rightarrow (\lambda-3)(\lambda^2+\lambda-2)=0 \Rightarrow \lambda = ...$	dM1
	$\lambda = 1, -2, (3)$	A1
		(3)
(ii)	$\begin{pmatrix}1 & -1 & 4\\ 3 & 2 & -1\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow \begin{matrix}x-y+4z=x\\ 3x+2y-z=y\\ 2x+y-z=z\end{matrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \cdots \left(\begin{pmatrix}-1\\4\\1\end{pmatrix}\right)$ or $\begin{pmatrix}1 & -1 & 4\\ 3 & 2 & -1\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = -2\begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow \begin{matrix}x-y+4z=-2x\\ 3x+2y-z=-2y\\ 2x+y-z=-2z\end{matrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \cdots \left(\begin{pmatrix}-1\\1\\1\end{pmatrix}\right)$	M1
	$\lambda=1 \rightarrow \begin{pmatrix}-1\\4\\1\end{pmatrix}$ or $\lambda=-2 \rightarrow \begin{pmatrix}-1\\1\\1\end{pmatrix}$	A1
	$\lambda=1 \rightarrow \begin{pmatrix}-1\\4\\1\end{pmatrix}$ and $\lambda=-2 \rightarrow \begin{pmatrix}-1\\1\\1\end{pmatrix}$	A1
		(3)
		Total 10

Notes

(a)

M1: Correct strategy to obtain the eigenvalue, multiplying the first row of matrix M with the given eigenvector, equating to λ and solving. Condone one slip for this mark but need to see some attempt at multiplying elements together.

A1: Correct eigenvalue obtained. Answer of 3 with no working scores both marks.

(b)

M1: Uses their eigenvalue from part (a) to form and solve 2 equations in *a* and *b* (must obtain a value for both)

A1: Correct values

(c)(i)

M1: Attempts to expand $\det(\mathbf{M}-\lambda\mathbf{I})$. This may be along any row or down any column, or via a "shoelace" approach (rule of Sarrus). Condone sign slips, but the overall structure should be correct. Working must be shown- do not accept solutions that just state the factorised form. The expansion must be seen in some form. Accept alternative approaches. Allow this mark to be scored if the attempt is seen under other parts of the question, or if *a* and *b* are not yet found.

Note: Attempts at using $\mathbf{Mx} = \lambda\mathbf{x}$ to obtain simultaneous equations are unlikely to make much progress

dM1: Sets the characteristic polynomial (which must be a four-term cubic) equal to zero (which may be implied) and solves via any valid method, including calculator, to find two other eigenvalues (not 3). This is a dependent mark, so as a minimum an attempt at the expansion must have been seen, but the cubic may not be seen fully simplified. If no working is shown, then their values must be correct for their unfactorised cubic (or apply general rules for a 3TQ if they factorise first)

A1: Correct remaining eigenvalues obtained

(ii)

M1: Applies $\mathbf{Mx} = \lambda\mathbf{x}$ with one of their calculated eigenvalues, however found (not using 3), forms simultaneous equations and proceeds to solve to find one eigenvector, or finds the cross product of any two rows of $\mathbf{M} - \lambda\mathbf{I}$ with their eigenvalues to obtain one eigenvector. Condone $x = \ldots$, $y = \ldots$ $z = \ldots$ for this mark.

A1: One correct eigenvector. Allow any multiple of the correct vector. Must be a vector.

A1: Both correct eigenvectors. Allow any multiples of the correct vectors. Must be a vector.