Question Number	Scheme	Marks
4.	$y = \text{arsinh}\, x + \text{arsinh}\left(\frac{1}{x}\right) \qquad x > 0$	
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+\frac{1}{x^2}}} \times -x^{-2}$	M1
	$= \frac{1}{\sqrt{1+x^2}} - \frac{1}{x^2\sqrt{1+\frac{1}{x^2}}} = \underline{\underline{\frac{1}{\sqrt{1+x^2}} - \frac{1}{x\sqrt{1+x^2}}}} = \frac{x-1}{x\sqrt{1+x^2}}$ * or $= \frac{1}{\sqrt{1+x^2}} - \frac{x^{-2}}{\sqrt{1+\frac{1}{x^2}}} = \underline{\underline{\frac{\sqrt{1+\frac{1}{x^2}} - x^{-2}\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{1+\frac{1}{x^2}}}}} = \frac{\sqrt{1+x^2} - \frac{1}{x}\sqrt{1+x^2}}{1+x^2} = \frac{1-\frac{1}{x}}{\sqrt{x^2+1}} = \frac{x-1}{x\sqrt{1+x^2}}$ *	A1*
		(2)
(b)	$\frac{x-1}{x\sqrt{1+x^2}} = 0 \Rightarrow x = 1$	B1
	$y = \text{arsinh}\, 1 + \text{arsinh}\left(\frac{1}{1}\right) = 2\,\text{arsinh}\, 1 = 2\ln\left(1+\sqrt{1^2+1}\right)$ o.e.	M1
	$y = \ln\left(3+2\sqrt{2}\right)$ or $\ln\left(1+\sqrt{2}\right)^2$ or $2\ln\left(1+\sqrt{2}\right)$	A1
		(3)
Alt for M1A1	$y = 2\,\text{arsinh}\, 1 \Rightarrow \sinh\frac{y}{2} = 1 \Rightarrow \frac{\mathrm{e}^{\frac{y}{2}} - \mathrm{e}^{-\frac{y}{2}}}{2} = 1 \Rightarrow \mathrm{e}^{y} - 2\mathrm{e}^{\frac{y}{2}} - 1 = 0$ $\Rightarrow \mathrm{e}^{\frac{y}{2}} = 1 \pm \sqrt{2} \Rightarrow \frac{y}{2} = \ldots$	M1
	$y = \ln\left(3+2\sqrt{2}\right)$ or $\ln\left(1+\sqrt{2}\right)^2$ or $2\ln\left(1+\sqrt{2}\right)$	A1
		Total 5

Notes

(a)

M1: Differentiates to the form $\frac{1}{\sqrt{1+x^2}}+\frac{kx^{-2}}{\sqrt{1+\frac{1}{x^2}}}$.

Note: There may be approaches involving e.g. $y=\text{ar}\sinh x \Rightarrow x=\sinh y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\cdots \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\cdots$ for one or both parts of the expression for *y*, but the attempt must reach an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the required form before the M mark can be awarded.

A1*: Correct proof with no errors and sufficient working shown. Must have at least the underlined steps shown above for the approach taken to score this mark.

(b)

B1: Deduces $x = 1$ when $\frac{\mathrm{d}y}{\mathrm{d}x}=0$

M1: Substitutes their value of *x* into the given equation and applies the logarithmic form of arsinh

A1: Correct value or equivalent e.g. $\ln\left(1+\sqrt{2}\right)^2$, $2\ln\left(1+\sqrt{2}\right)$ but do not condone poor bracketing here.

ALT for final two marks

M1: Substitutes their value of *x* into the given equation then uses the exponential form for sinh to form and solve a quadratic in $\mathrm{e}^{\frac{y}{2}}$ (usual rules) reaching as far as a value for $\frac{y}{2}$

A1: Correct value or equivalent e.g. $\ln\left(1+\sqrt{2}\right)^2$, $2\ln\left(1+\sqrt{2}\right)$ but do not condone poor bracketing here. ISW once a correct answer of the correct form is seen.