Question Number	Scheme	Marks
5(a)	$\int_1^6 x^n (3x-2)^{-\frac{1}{2}}\,\mathrm{d}x = \left[\frac{2}{3}x^n(3x-2)^{\frac{1}{2}}\right]_{(1)}^{(6)} - \frac{2}{3}\int_{(1)}^{(6)} nx^{n-1}(3x-2)^{\frac{1}{2}}\,\mathrm{d}x$	M1A1
	$= \ldots - \frac{2}{3}\int_{(1)}^{(6)} nx^{n-1}(3x-2)(3x-2)^{-\frac{1}{2}}\,\mathrm{d}x$	**d**M1
	$= \frac{2}{3}\times 6^n \times 4 - \frac{2}{3}\times 1 - \frac{2}{3}n\times 3I_n + \frac{4}{3}nI_{n-1}$ $\left(I_n = \frac{8}{3}\times 6^n - \frac{2}{3} - 2nI_n + \frac{4}{3}nI_{n-1}\right)$	**dd**M1
	$3I_n = 8\times 6^n - 2 - 6nI_n + 4nI_{n-1}$ $\Rightarrow (3+6n)I_n = 4nI_{n-1} + 8\times 6^n - 2*$	A1*
		(5)
(b)	$I_0 = \int_1^6 (3x-2)^{-\frac{1}{2}}\,\mathrm{d}x = \left[\frac{2}{3}(3x-2)^{\frac{1}{2}}\right]_1^6 = \frac{8}{3} - \frac{2}{3} = 2$	B1
	$21I_3 = 12I_2 + 1728 - 2$ $\left(\text{Implied by } 21I_3 = \frac{9966}{5} \text{ or } I_3 = \frac{3322}{35}\right)$ Or $15I_2 = 8I_1 + 288 - 2$ $\left(\text{Implied by } 15I_2 = 334 \text{ or } I_2 = \frac{334}{15}\right)$ Or $9I_1 = 4I_0 + 48 - 2$ $\left(\text{Implied by } 9I_1 = 54\right)$	M1
	$21I_3 = 12\left(\frac{8}{15}I_1 + \frac{286}{15}\right) + 1728 - 2 = 12\left(\frac{8}{15}\left(\frac{4}{9}I_0 + \frac{46}{9}\right) + \frac{286}{15}\right) + 1726 = \cdots \left(\frac{9966}{5}\right)$ or $= \frac{12}{15}(8I_1 + 288 - 2) + 1726 = \frac{32}{45}(4I_0 + 48 - 2) + \frac{9774}{5} = \cdots \left(\frac{9966}{5}\right)$	M1
	$I_3 = \frac{3322}{35}$	A1
		(4)
		Total 9

Notes

(a)

M1: Integrates by parts to obtain $\left[\alpha x^n (3x-2)^{\frac{1}{2}}\right]_{(1)}^{(6)} - \beta \int_{(1)}^{(6)} x^{n-1}(3x-2)^{\frac{1}{2}}\,\mathrm{d}x$ with or without limits

A1: Correct expression (limits not required on either part, and 'dx' may be missing)

dM1: Writes $(3x-2)^{\frac{1}{2}}$ as $(3x-2)(3x-2)^{-\frac{1}{2}}$ **Requires previous M mark.**

ddM1: Splits the integral, applies the given limits to the first expression and introduces I_n and $I_{n\text{-}1}$
Requires both previous M marks

A1*: Completes the proof with no mathematical errors seen to obtain the printed answer. Allow recovery of poor bracketing. Limits need not be shown on the integrals. Loss and subsequent recovery of the occasional 'n' and/or 'dx' loses this mark.

(b)

B1: Correct value for I_0 This may be implied by a correct value for I_1, I_2 or I_3

M1: Applies the reduction formula **at least once** to obtain I_3 in terms of I_2 or I_2 in terms of I_1 or I_1 in terms of I_0 Allow slips but must have the correct number of terms. May be embedded in another expression for I_1, I_2 or I_3 This could be implied by the exact numerical values seen coupled with an obvious attempt. Allow a multiple of I_3, I_2, I_1 e.g. $21I_3 = \ldots$

M1: Completes the process to obtain I_3 in terms of I_0 (or I_1 if I_1 is found by integration) and substitutes for I_0 (or I_1) to find a value for kI_3 or I_3

A1: For $\frac{3322}{35}$ or exact equivalent

Note: It is possible for I_1 to be found using integration, and a correct value would imply B1, but method marks are for using the reduction formula.