Question Number	Scheme	Marks
7(a)	$y=\cos 2x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x$ and then also: $S=2\pi\displaystyle\int_{(0)}^{\left(\frac{\pi}{4}\right)} y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,(\mathrm{d}x)$ and $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\ldots\Rightarrow S=\cdots$ or $S=2\pi\displaystyle\int_{(0)}^{\left(\frac{\pi}{4}\right)} \cos 2x\sqrt{1+\left(-2\sin 2x\right)^2}\,(\mathrm{d}x)\Rightarrow S=\cdots$	M1
	$S=2\pi\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$*	A1*
		(2)
(b)	$2\sin 2x=\sinh\theta\Rightarrow 4\cos 2x\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\cosh\theta$ o.e.	B1
	$S=2\pi\displaystyle\int \cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x=2\pi\int \cancel{\cos 2x}\sqrt{1+\sinh^2\theta}\,\dfrac{\cosh\theta}{\cancel{4\cos 2x}}\mathrm{d}\theta$	M1
	$=\dfrac{\pi}{2}\displaystyle\int \cosh\theta\sqrt{1+\sinh^2\theta}\,\mathrm{d}\theta=\frac{\pi}{2}\int \cosh^2\theta\,\mathrm{d}\theta$	A1
ALT for first 3 marks	$x=\dfrac{1}{2}\arcsin\left(\dfrac{1}{2}\sinh\theta\right)\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\dfrac{1}{2}\dfrac{\frac{1}{2}\cosh\theta}{\sqrt{1-\frac{1}{4}\sinh^2\theta}}$	$\theta=\operatorname{arsinh}(2\sin 2x)\Rightarrow\dfrac{\mathrm{d}\theta}{\mathrm{d}x}=\dfrac{4\cos 2x}{\sqrt{1+4\sin^2 2x}}$ B1
	$S=2\pi\displaystyle\int \cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ $=2\pi\displaystyle\int \cancel{\sqrt{1-\frac{1}{4}\sinh^2\theta}}\sqrt{1+\sinh^2\theta}\,\frac{1}{2}\frac{\frac{1}{2}\cosh\theta}{\cancel{\sqrt{1-\frac{1}{4}\sinh^2\theta}}}\mathrm{d}\theta$	$S=2\pi\displaystyle\int \cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ $=2\pi\displaystyle\int \cancel{\cos 2x}\sqrt{1+4\sin^2 2x}\,\dfrac{\sqrt{1+4\sin^2 2x}}{\cancel{4\cos 2x}}\mathrm{d}\theta$ $=2\pi\displaystyle\int \cancel{\cos 2x}\,\dfrac{1+\sinh^2\theta}{\cancel{4\cos 2x}}\mathrm{d}\theta$ M1
	$=\dfrac{\pi}{2}\displaystyle\int \cosh^2\theta\,\mathrm{d}\theta$	A1
	$=\dfrac{\pi}{2}\displaystyle\int\left(\frac{1}{2}+\frac{1}{2}\cosh 2\theta\right)\mathrm{d}\theta$ **or** $=\dfrac{\pi}{2}\displaystyle\int\left(\frac{\mathrm{e}^{\theta}+\mathrm{e}^{-\theta}}{2}\right)^2\mathrm{d}\theta=\frac{\pi}{2}\int\left(\frac{\mathrm{e}^{2\theta}+2+\mathrm{e}^{-2\theta}}{4}\right)\mathrm{d}\theta$	M1
	$=\dfrac{\pi}{4}\left[\theta+\dfrac{1}{2}\sinh 2\theta\right]$ **or** $\dfrac{\pi}{2}\left[\dfrac{1}{8}\mathrm{e}^{2\theta}+\dfrac{1}{2}\theta-\dfrac{1}{8}\mathrm{e}^{-2\theta}\right]$	A1
	$\dfrac{\pi}{4}\left[\theta+\dfrac{1}{2}\sinh 2\theta\right]_0^{\operatorname{arsinh}2}=\dfrac{\pi}{4}\left(\operatorname{arsinh}2+\dfrac{1}{2}\sinh\left(2\operatorname{arsinh}2\right)\right)$ **or** $\dfrac{\pi}{2}\left[\dfrac{1}{8}\mathrm{e}^{2\theta}+\dfrac{1}{2}\theta-\dfrac{1}{8}\mathrm{e}^{-2\theta}\right]_0^{\operatorname{ar\,sinh}2}=\dfrac{\pi}{2}\left(\dfrac{1}{8}\mathrm{e}^{2\ln\left(2+\sqrt{5}\right)}+\dfrac{1}{2}\ln\left(2+\sqrt{5}\right)-\dfrac{1}{8}\mathrm{e}^{-2\ln\left(2+\sqrt{5}\right)}\right)$	M1
	$=\dfrac{\pi}{4}\left(\ln\left(2+\sqrt{5}\right)+2\sqrt{5}\right)$	A1
		(7)
		Total 9

Notes

(a)

Note: for part (a) there are two elements that must be combined to score the M mark

- Differentiation of *y* to the correct form
- Showing the substitution of their derivative and *y* into a correct formula (which could be implied by a derivative of the correct form, a correct $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ for their derivative and a correct SA formula quoted).

M1: Differentiates *y* to obtain $y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=k\sin 2x$ and substitutes into the correct surface area formula.

Must see evidence of using the correct formula, which could be implied by a correct substitution with their derivative and *y* substituted- simply stating the result after differentiation (even if correct differentiation) is M0A0 without some evidence of application of their derivative to the correct SA formula. Differentiation must be seen, and the 2π but condone the omission of the limits and/or the $\mathrm{d}x$ for this mark.

A1*: Reaches the printed answer with no errors or omissions. Fully correct working shown, including limits, and the $\mathrm{d}x$ etc. Must be fully shown. Some exemplars are below:

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ **M0A0***

(Correct derivative but no substitution shown, no use of formula seen, states given answer with no justification)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+\left(-2\sin 2x\right)^2}\,\mathrm{d}x=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ **M1A1***

(Correct derivative, use of correct formula seen - implied by a correct substitution with their derivative and *y*)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ **M0A0***

(Incorrect derivative, no use of formula seen by correct substitution, states given result with no justification)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+\left(2\sin 2x\right)^2}\,\mathrm{d}x=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ **M1A0***

(Incorrect derivative, use of correct formula implied by a correct substitution with their derivative and *y*)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x$ and $S=2\pi\int_0^{\frac{\pi}{4}}y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$ **M0A0***

(Correct derivative and formula seen but no evidence of using the correct formula to obtain the given result)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x$ and $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4\sin^2 2x \Rightarrow S=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$

M0A0*

(Correct derivative and square seen but no evidence of using this in the correct formula to obtain the given result)

$y=\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 2x,\ \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4\sin^2 2x,\ S=2\pi\int_0^{\frac{\pi}{4}}y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=2\pi\int_0^{\frac{\pi}{4}}\cos 2x\sqrt{1+4\sin^2 2x}\,\mathrm{d}x$

M1A1*

(Correct derivative and square seen and use of this in the correct formula is implied by the correct formula being quoted.

(b)

B1: Correct differentiation. Accept equivalent forms if the substitution is rearranged first. Must be a fully correct differentiation to score this mark.

M1: Substitutes fully into the given integral and makes progress to eliminate the *x* terms. Must substitute fully, including the $\mathrm{d}x$ and arrive at an integral involving only θ. Limits not needed for this mark and condone if they are not changed yet.

A1: Correct integral in terms of $\cosh\theta$ (can ignore limits for this mark).

M1: Applies $\cosh^2\theta = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2\theta$ or uses the correct exponential form and expands to obtain an integrable form for the integral (can ignore limits for this mark)

A1: Fully correct integration (can ignore limits for this mark)

M1: Applies the limits 0 and arsinh 2 or e.g. 0 and $\ln\left(2+\sqrt{5}\right)$ to an integrated expression of the form $a\theta + b\sinh 2\theta$ or $a\mathrm{e}^{2\theta} + b\theta + c\mathrm{e}^{-2\theta}$. Must see clear evidence of the limits being applied, but condone the omission of the lower limit as it gives 0

A1: Cao. Must be in the exact form requested.