Question Number	Scheme	Marks
	Accept **i, j, k** notation or column vector notation throughout this question	
8(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -6 \\ -3 & 1 & 2 \end{vmatrix} = \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix}$	B1
		(1)
(b)	$\begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$ or $\begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
ALT Using another point on the plane	e.g. $\lambda = 0, \mu = 1 \Rightarrow$ point $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ on the plane $\Rightarrow \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$ or $\Rightarrow \begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & 11 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix}$ Or Points on the line are (see below) $\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ so direction is $\begin{pmatrix} \frac{52}{3} \\ 4 \\ -\frac{40}{3} \end{pmatrix}$ or any multiple e.g. $\begin{pmatrix} 52 \\ 12 \\ -40 \end{pmatrix}$ etc	M1
	$x = \ldots(0) \Rightarrow \begin{cases} 2y+11z = 25 \\ -y+z = 7 \end{cases} \Rightarrow z = \ldots(3), y = \ldots(-4)$ $y = \ldots(0) \Rightarrow \begin{cases} 8x+11z = 25 \\ x+z = 7 \end{cases} \Rightarrow z = \ldots\left(-\frac{31}{3}\right), x = \ldots\left(\frac{52}{3}\right)$ $z = \ldots(0) \Rightarrow \begin{cases} 8x+2y = 25 \\ x-y = 7 \end{cases} \Rightarrow x = \ldots\left(\frac{39}{10}\right), y = \ldots\left(-\frac{31}{10}\right)$ **Note**: points will have the form $\left(\frac{52+13\alpha}{3}, \alpha, \frac{-31-10\alpha}{3}\right)$	M1

	Common points $(0, -4, 3)\ \left(\frac{52}{3}, 0, -\frac{31}{3}\right)\ \left(\frac{39}{10}, -\frac{31}{10}, 0\right)\ \left(1, -\frac{49}{13}, \frac{29}{13}\right)\ \left(\frac{65}{3}, 1, -\frac{41}{3}\right)\left(\frac{13}{5}, -\frac{17}{5}, 1\right)$	
	$\left(\mathbf{r}-\left(-4\mathbf{j}+3\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)
Alt to find a point and/or direction vector for either M1 mark	Combining cartesian equations together (other eliminations are possible): $\begin{cases} x-y+z=7 \\ 8x+2y+11z=25 \end{cases} \Rightarrow 3x-13y=52 \Rightarrow y=\frac{3x-52}{13} \quad x=\frac{52+13y}{3}$ $z=7-x+y=\frac{-31-10y}{3} \Rightarrow y=\frac{-3z-31}{10}$ So $\frac{3x-52}{13}=y=\frac{-3z-31}{10} \Rightarrow \frac{x-\frac{52}{3}}{\frac{13}{3}}=\frac{y-0}{1}=\frac{z+\frac{31}{3}}{-\frac{10}{3}}$ so point is $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ Or Direction is $\begin{pmatrix} \frac{13}{3} \\ 1 \\ -\frac{10}{3} \end{pmatrix}$ or any multiple	M1
	point is $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ and direction is $\begin{pmatrix} \frac{13}{3} \\ 1 \\ -\frac{10}{3} \end{pmatrix}$ or any multiple	M1
	$\left(\mathbf{r}-\left(-4\mathbf{j}+3\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)

(d)	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\13&3&-10\\1&-1&-1\end{vmatrix}=\begin{pmatrix}-13\\3\\-16\end{pmatrix}$ or $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-1&-1\\13&3&-10\end{vmatrix}=\begin{pmatrix}13\\-3\\16\end{pmatrix}$	M1
	$d=\left\|\frac{\left(-4\mathbf{j}+3\mathbf{k}-\left(3\mathbf{i}+2\mathbf{k}\right)\right)\square\left(-13\mathbf{i}+3\mathbf{j}-16\mathbf{k}\right)}{\sqrt{13^2+3^2+16^2}}\right\|=...$	M1
	$d=\frac{11}{\sqrt{434}}$	A1
		(4)
Alt using general points on the lines	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	Let the general points on the lines be X and Y $\overrightarrow{XY}=\begin{pmatrix}2+\lambda\\1-\lambda\\3-\lambda\end{pmatrix}-\begin{pmatrix}0+13\mu\\-4+3\mu\\3-10\mu\end{pmatrix}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\Rightarrow$ $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}13\\3\\-10\end{pmatrix}=0$ and $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-1\end{pmatrix}=0\Rightarrow\begin{cases}41+20\lambda-278\mu=0\\-3+3\lambda-20\mu=0\end{cases}$ $\Rightarrow\lambda=...\left(\frac{827}{217}\right),\mu=...\left(\frac{183}{434}\right)$	M1
	$\overrightarrow{XY}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}=\begin{pmatrix}\frac{143}{434}\\-\frac{33}{434}\\\frac{88}{217}\end{pmatrix}$ $\left\|\overrightarrow{XY}\right\|=\sqrt{\left(\frac{143}{434}\right)^2+\left(\frac{33}{434}\right)^2+\left(\frac{88}{217}\right)^2}=...\left(\frac{121}{434}\right)$	M1
	$d=\frac{11}{\sqrt{434}}$	A1
		(4)
		Total 10

Notes

(a)

B1: Correct vector found. ISW once a correct vector product is seen e.g. if the components are divided by 2 etc.

(b)

M1: Attempts $\mathbf{r} \bullet \mathbf{n} =$ p form for the plane by finding the scalar product between their part (a) or a multiple of their part (a) and $4\mathbf{i}+2\mathbf{j}-\mathbf{k}$ (or another point on the line, correctly obtained by substituting values of the parameters into the line) . If the scalar product is found with a point not on the line, then M0.

A1: Correct equation

(c)

M1: Attempts the vector product between $8\mathbf{i}+2\mathbf{j}+11\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$ or e.g. finds 2 points on the line and subtracts to find the direction. If method not directly shown, then two correct components correct implies method.

M1: Attempts to find a point on the line of intersection, either by substituting a value for *x*, *y*, *z* into the cartesian equations of the planes, and solving to find the other two values or by combining the cartesian equations together. M0 if they find a correct point then multiply by a scalar.

A1: Any correct equation in the required form e.g. $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times\left(-\frac{13}{10}\mathbf{i}-\frac{3}{10}\mathbf{j}+\mathbf{k}\right)=\mathbf{0}$

ALT

M1: Combines the cartesian equations together by eliminating one variable and finding one in terms of the other two or expressing two variables in terms of the other one, and making progress to write the line in the form $\frac{x-a}{d}=\frac{y-b}{e}=\frac{z-c}{f}$ and then correctly extracting either their point or their direction correctly from their line equation. Allow minor slips if the line is in the correct form.

M1: Correctly extracts both the point and the direction from their line equation in cartesian form. Allow minor slips if the line is in the correct form.

A1: Forms the correct equation of the line in the correct form. Condone the '**0**' and/or the '***r***' without an underscore to indicate that it is a vector.

(d)

M1: Attempts the direction of l_2

M1: Attempts the vector product between their direction from part (c) and their direction of l_2. If method not directly shown, then two correct components implies the correct method.

M1: A full method to find the required distance using formula $\left|\frac{(\mathbf{a}-\mathbf{c})\bullet\mathbf{n}}{|\mathbf{n}|}\right|$ where $\mathbf{a}$ and $\mathbf{c}$ are the position vectors of individual points on the lines l_1 and l_2 and $\mathbf{n}$ is the vector product of the direction vectors for their lines. $\left(\text{e.g. } \mathbf{r}_1=\begin{pmatrix}0\\-4\\3\end{pmatrix}+\lambda\begin{pmatrix}13\\3\\-10\end{pmatrix} \text{ and } \mathbf{r}_2=\begin{pmatrix}3\\0\\2\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-1\end{pmatrix}\right)$. Note that the point *A* could also be used in the line $\mathbf{r}_2$ so that $\begin{pmatrix}3\\0\\2\end{pmatrix}$ is replaced with $\begin{pmatrix}2\\1\\3\end{pmatrix}$ (or another point). The numerator may be seen as a scalar triple product e.g. $\begin{vmatrix}-3&-4&1\\13&3&-10\\1&-1&-1\end{vmatrix}$ etc.

A1: For $\frac{11}{\sqrt{434}}$ or awrt 0.528

ALT

M1: Attempts the direction of l_2

M1: Forms the general point between the two lines, attempts the dot product between the general point and both direction vectors of the lines l_1 and l_2 to form and then solve two simultaneous equations in their parameters. Must find a value for both.

M1: Substitutes their parameters to find their $\overrightarrow{XY}$ and then finds the magnitude of their $\overrightarrow{XY}$

A1: For $\frac{11}{\sqrt{434}}$ or awrt 0.528