8: The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(1)

(b) Hence show that the equation of Π_1 can be written in the form

$$\mathbf{r}.(8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where p is a constant to be determined.

(2)

Given that

- the plane Π_2 has equation $x - y + z = 7$
- the planes Π_1 and Π_2 intersect in the line l_1

(c) determine an equation for l_1 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(3)

Given also that

- the point A has coordinates (2, 1, 3)
- the point B has coordinates (3, 0, 2)
- the line l_2 passes through A and B

(d) determine the shortest distance between l_1 and l_2

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 10 marks)