Question Number	Scheme	Marks
9(a)	$b^2 = a^2\left(e^2-1\right) \Rightarrow 49 = 64\left(e^2-1\right) \Rightarrow e^2 = \ldots$	M1
	$49 = 64e^2 - 64 \Rightarrow e^2 = \dfrac{113}{64} \Rightarrow e = \dfrac{\sqrt{113}}{8}$ *	A1*
		(2)
(b)	$x = 8\sec t, y = 7\tan t \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{7\sec^2 t}{8\sec t\tan t} = \dfrac{7\sec^2\theta}{8\sec\theta\tan\theta}\ \left(= \dfrac{7}{8}\operatorname{cosec}\theta\right)$ (Parametric diff.) or $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1 \Rightarrow \dfrac{x}{32} - \dfrac{2y}{49}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{49x}{64y} = \dfrac{7\sec\theta}{8\tan\theta}$ (Implicit diff.) or $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1 \Rightarrow y = 7\sqrt{\dfrac{x^2}{64}-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{7}{2}\left(\dfrac{x^2}{64}-1\right)^{-\frac{1}{2}} \times \dfrac{x}{32} = \dfrac{7}{2}\left(\sec^2\theta - 1\right)^{-\frac{1}{2}} \times \dfrac{8\sec\theta}{32}$	B1
	$y - 7\tan\theta = \dfrac{7\sec\theta}{8\tan\theta}\left(x - 8\sec\theta\right)$ or $'y = mx + c' \Rightarrow 7\tan\theta = \dfrac{7\sec\theta}{8\tan\theta} \times 8\sec\theta + c \Rightarrow c = \ldots\left(-\dfrac{7}{\tan\theta}\right)$	M1
	$y - 7\tan\theta = \dfrac{7\sec\theta}{8\tan\theta}\left(x - 8\sec\theta\right) \Rightarrow 8y\tan\theta - 56\tan^2\theta = 7x\sec\theta - 56\sec^2\theta$ $\Rightarrow \dfrac{x}{8}\sec\theta - \dfrac{y}{7}\tan\theta = \sec^2\theta - \tan^2\theta$ $\Rightarrow \dfrac{x}{8}\sec\theta - \dfrac{y}{7}\tan\theta = 1$*	A1*
		(3)
(c)	$\left(0, -7\cot\theta\right)$ o.e.	B1
		(1)
(d)	$\left(0, \dfrac{113}{7}\tan\theta\right)$ o.e.	B1
		(1)
(e)	Centre is $\left(0, \dfrac{113}{14}\tan\theta - \dfrac{7}{2}\cot\theta\right)$	B1
	(Radius is) $\dfrac{113}{14}\tan\theta + \dfrac{7}{2}\cot\theta$	B1
	$x^2 + \left(y - \left(\dfrac{113}{14}\tan\theta - \dfrac{7}{2}\cot\theta\right)\right)^2 = \left(\dfrac{113}{14}\tan\theta + \dfrac{7}{2}\cot\theta\right)^2$	M1
	$y = 0 \Rightarrow x^2 + \left(\dfrac{113}{14}\tan\theta - \dfrac{7}{2}\cot\theta\right)^2 = \left(\dfrac{113}{14}\tan\theta + \dfrac{7}{2}\cot\theta\right)^2$ $\Rightarrow x^2 = 2\times 2\times\dfrac{113}{14}\tan\theta\times\dfrac{7}{2}\cot\theta = 113 \Rightarrow x = \pm\sqrt{113}$ Foci are at $\left(\pm ae, 0\right) = \left(\pm 8\times\dfrac{\sqrt{113}}{8}, 0\right) = \left(\pm\sqrt{113}, 0\right)$	M1
	Hence the circle passes through the foci of H *	A1*
		(5)

ALT for final M1A1	Foci are at $(\pm ae, 0) = \left(\pm 8 \times \frac{\sqrt{113}}{8}, 0\right) = \left(\pm\sqrt{113}, 0\right)$ Substituting into the circle equation: $LHS = \left(\pm\sqrt{113}\right)^2 + \left(\frac{113}{14}\right)^2 \tan^2\theta + \left(\frac{7}{2}\right)^2 \cot^2\theta - 2\left(\frac{113}{14}\right)\left(\frac{7}{2}\right) = \left(\frac{113}{14}\right)^2 \tan^2\theta + \left(\frac{7}{2}\right)^2 \cot^2\theta + \frac{113}{2}$ $RHS = \left(\frac{113}{14}\right)^2 \tan^2\theta + \left(\frac{7}{2}\right)^2 \cot^2\theta + 2\left(\frac{113}{14}\right)\left(\frac{7}{2}\right) = \left(\frac{113}{14}\right)^2 \tan^2\theta + \left(\frac{7}{2}\right)^2 \cot^2\theta + \frac{113}{2}$	M1
	$LHS = RHS$ so the circle passes through the foci of H	A1*
ALT 2 Circle theorems and gradients	$m_{FR} = \frac{\frac{113}{7}\tan\theta - 0}{0 - \sqrt{113}} = -\frac{113}{7\sqrt{113}}\tan\theta$ Or $m_{FR} = \frac{\frac{113}{7}\tan\theta - 0}{0 + \sqrt{113}} = \frac{113}{7\sqrt{113}}\tan\theta$	B1
	$m_{FQ} = \frac{-7\cot\theta - 0}{0 - \sqrt{113}} = \frac{7}{\sqrt{113}}\cot\theta$ Or $m_{FQ} = \frac{-7\cot\theta - 0}{0 + \sqrt{113}} = -\frac{7}{\sqrt{113}}\cot\theta$	B1
	$m_{FR} \times m_{FQ} = -\frac{113}{7\sqrt{113}}\tan\theta \times \frac{7}{\sqrt{113}}\cot\theta = -1$ Or $m_{FR} \times m_{FQ} = \frac{113}{7\sqrt{113}}\tan\theta \times -\frac{7}{\sqrt{113}}\cot\theta = -1$	M1
	For both $m_{FR} \times m_{FQ} = -\frac{113}{7\sqrt{113}}\tan\theta \times \frac{7}{\sqrt{113}}\cot\theta = -1$ and $m_{FR} \times m_{FQ} = \frac{113}{7\sqrt{113}}\tan\theta \times -\frac{7}{\sqrt{113}}\cot\theta = -1$	M1
	$m_{FR} \times m_{FQ} = -1$ for both foci, so the circle passing through F has QR as a diameter	A1*
		(5)

ALT 3 Circle theorems and Pythagoras	$FQ = \sqrt{113 + 49\cot^2\theta}$ or $FQ^2 = 113 + 49\cot^2\theta$	B1
	$FR = \sqrt{113 + \left(\frac{113}{7}\tan\theta\right)^2}$ or $FR^2 = 113 + \left(\frac{113}{7}\tan\theta\right)^2$ or better	B1
	$FQ^2 + FR^2 = 49\cot^2\theta + \left(\frac{113}{7}\tan\theta\right)^2 + 226$	M1
	$QR^2 = \left(7\cot\theta + \frac{113}{7}\tan\theta\right)^2 = 49\cot^2\theta + \left(\frac{113}{7}\tan\theta\right)^2 + 226$	M1
	$QR^2 = FR^2 + FQ^2$ for both foci, so the circle passing through F has QR as a diameter	A1*
		(5)
		Total 12

Notes

(a)

M1: Uses a correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ and rearranges for e^2

A1*: Correct proof with sufficient working and no errors. Must have at least one line of correct working between their correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ substituted before the printed answer. It's not sufficient to go from a correct formula with values substituted directly to a simplified e^2 or e without some intermediate working. Do not accept $e = \pm\frac{\sqrt{113}}{8}$. Some exemplars are below

$b^2 = a^2\left(e^2 - 1\right) \Rightarrow 49 = 64\left(e^2 - 1\right) \Rightarrow e = \frac{\sqrt{113}}{8}$ **M1A0*** (no intermediate line of working shown)

$49 = 64\left(e^2 - 1\right) \Rightarrow e^2 = \frac{113}{64} \Rightarrow e = \frac{\sqrt{113}}{8}$ **M1A0*** (no intermediate line of working shown)

$49 = 64\left(e^2 - 1\right) \Rightarrow e^2 = \frac{49 + 64}{64} \Rightarrow e = \frac{\sqrt{113}}{8}$ **M1A1*** (Intermediate line of working before a simplified e^2 or e is seen).

$7^2 = 8^2\left(e^2 - 1\right) \Rightarrow e^2 - 1 = \frac{49}{64} \Rightarrow e = \frac{\sqrt{113}}{8}$ **M1A1*** (Intermediate line of working before a simplified e^2 or e is seen).

Note: accept a verification method which shows that 'LHS = RHS' after a correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ is seen.

(b)

B1: Correct tangent gradient obtained using calculus. Need not be simplified for this mark, but it must be a fully correct derivative expression in terms of θ.

M1: Uses a correct straight-line method with their gradient and the point P. If they are using $y = mx + c$ then they must make progress to finding a value for their 'c'.

A1*: Correct proof with no errors and sufficient working shown. At least one line of intermediate working required between their correct line equation and the given answer.

(c)

B1: Correct coordinates or $x = 0$ and $y = -7\cot\theta$ o.e. e.g. $\left(0, -\frac{7}{\tan\theta}\right)$ or $\left(0, -\frac{7\cos\theta}{\sin\theta}\right)$. Can award if seen under other parts of the question, but must be clearly labelled as point Q.

(d)

B1: Correct coordinates or $x = 0$ and $y = \frac{113}{7}\tan\theta$ o.e. e.g. $\left(0, \frac{113\sin\theta}{7\cos\theta}\right)$ or $\left(0, \frac{113}{7\cot\theta}\right)$. Can award if seen under other parts of the question, but must be clearly labelled as point *R*.

(e)

B1: Correct centre in any equivalent form e.g.

$\left(0, \frac{113\tan\theta - 49\cot\theta}{14}\right)$ or $\left(0, \frac{113\frac{\sin\theta}{\cos\theta} - 49\frac{\cos\theta}{\sin\theta}}{14}\right)$ or $\left(0, \frac{113\sin^2\theta - 49\cos^2\theta}{14\sin\theta\cos\theta}\right)$

B1: Correct radius in any equivalent form e.g. $\frac{113\sin\theta}{14\cos\theta} + \frac{7\cos\theta}{2\sin\theta}$ or $\frac{113\sin^2\theta + 49\cos^2\theta}{14\sin\theta\cos\theta}$ or $\frac{64\sin^2\theta + 49}{14\sin\theta\cos\theta}$

Ignore the labelling for this mark. Award this mark if the correct expression is seen.

M1: Uses their centre and radius to form the equation of the circle in the form $x^2 + (y - \cdots)^2 = r^2$

M1: Substitutes $y = 0$, solves for x and uses the result from part (a) to compare with the values of $\pm ae$

A1*: Fully correct work with conclusion

ALT for final M1A1

M1: Substitutes the correct foci into their circle equation, and makes some attempt to simplify both sides to show they are equal- may be seen with *x* squared which is fine.

A1: Shows that 'LHS = RHS' for their correct circle equation and gives a basic conclusion.

ALT 2 using circle theorems and gradients

B1: for finding the gradient of *FR* for either focus *F*

B1: For finding the gradient of *FQ* for either focus *F*

M1: For using $m_{FR} \times m_{FQ} = -1$ to show that *FQ* is perpendicular to *FR* for either focus

M1: For showing that *FQ* is perpendicular to *FR* for both foci

A1: Concludes that the circle passing through the focus (point *F*) has *QR* as a diameter

ALT 3 using circle theorems and converse of Pythagoras

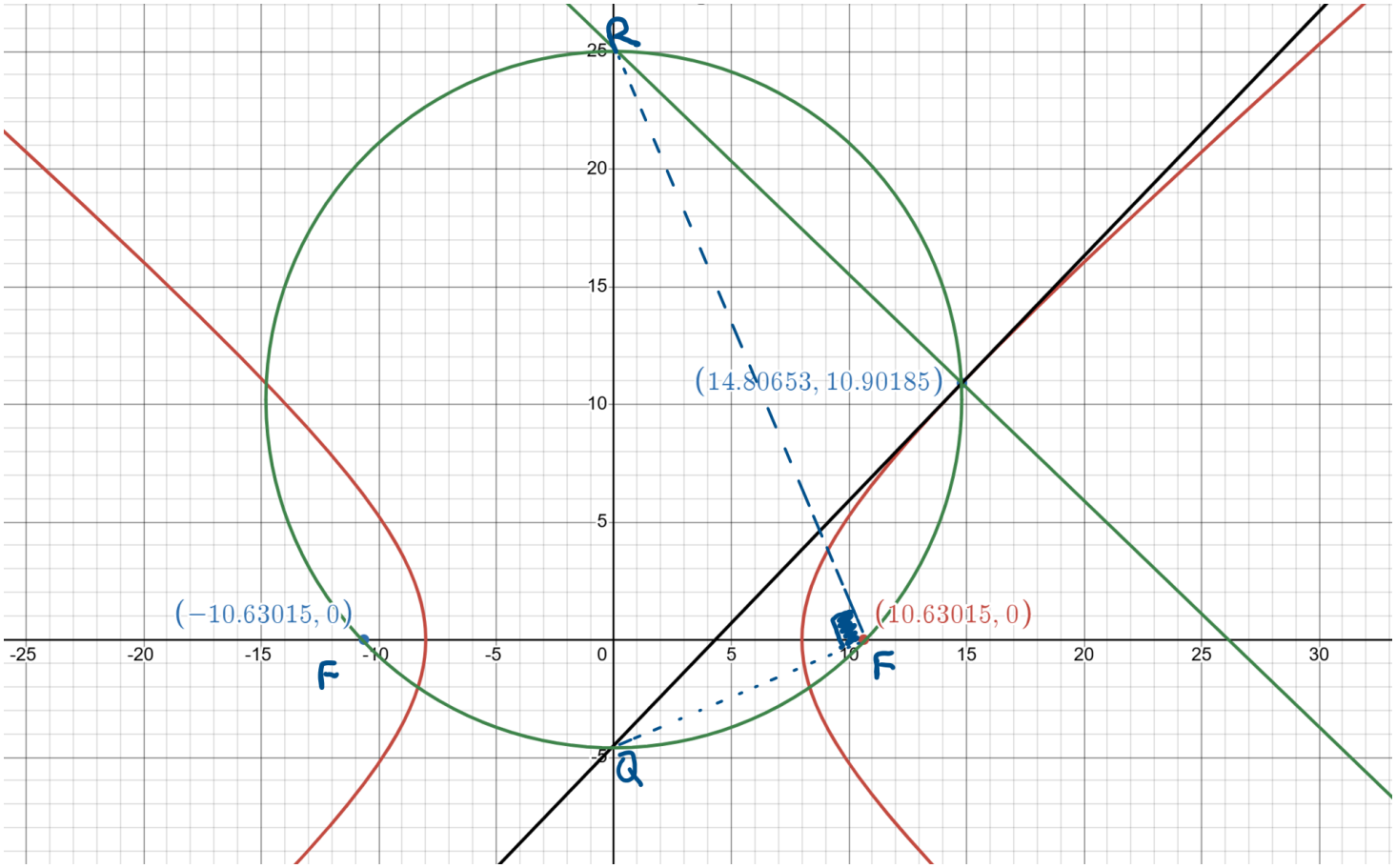
B1: For finding FQ or FQ^2

B1: FR or FR^2

M1: For finding $FQ^2 + FR^2$

M1 For showing that $QR^2 = FQ^2 + FR^2$

A1: Concludes that the circle passing through the focus (point *F*) has *QR* as a diameter


R
Q
F
F
(14.80653, 10.90185)
(−10.63015, 0)
(10.63015, 0)
25
20
15
10
5
-5
-25
-20
-15
-10
-5
0
5
10
15
20
25
30

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