9: The hyperbola H has equation $\frac{x^2}{64} - \frac{y^2}{49} = 1$ and eccentricity e.

(a) Show that $e = \frac{\sqrt{113}}{8}$

(2)

The point $(8\sec\theta, 7\tan\theta)$, where $0 < \theta < \frac{\pi}{2}$, lies on H.

(b) Use calculus to show that the tangent to H at P has equation

$$\frac{x}{8}\sec\theta - \frac{y}{7}\tan\theta = 1$$

(3)

The tangent to H at P meets the y-axis at the point Q.

(c) Write down the coordinates of Q.

(1)

The normal to H at P

- has equation $8x\cos\theta + 7y\cot\theta = 113$
- meets the y-axis at the point R

(d) Write down the coordinates of R.

(1)

(e) Hence show that the circle with QR as a diameter passes through the foci of H.

(5)

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Question 9 continued

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(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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