Question	Scheme	Marks
5(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2\sinh 2t \text{ and } \frac{\mathrm{d}y}{\mathrm{d}t} = 4\cosh t$	**B1**
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 4\sinh^2 2t + 16\cosh^2 t$	**M1**
	$= 4(2\sinh t\cosh t)^2 + 16\cosh^2 t$ $= 4(\cosh^2 2t - 1) + 16\cosh^2 t$ $= 4\left(2\cosh^2 t - 1\right)^2 - 4 + 16\cosh^2 t$	**M1**
	$= 16\cosh^2 t\left(\sinh^2 t + 1\right)$ $= 16\cosh^4 t - 16\cosh^2 t + 4 - 4 + 16\cosh^2 t$	**M1**
	$= 16\cosh^4 t$	**A1**
		(5)
(b)	$s = \int_0^{\frac{3}{2}} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t = \int_0^{\frac{3}{2}} 4\cosh^2 t\,\mathrm{d}t = \int_0^{\frac{3}{2}} 2\left(1+\cosh 2t\right)\mathrm{d}t$	**M1**
	$= \left[2t + \sinh 2t\right]_0^{\frac{3}{2}} = \left(3 + \sinh 3\right) - \left(0 + 0\right)$	**dM1**
	$= 3 + \sinh 3$	**A1**
		(3)
(c)	$S_x = 2\pi\int_0^{\frac{3}{2}} y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t = 2\pi\int_0^{\frac{3}{2}} 4\sinh t \times 4\cosh^2 t\,\mathrm{d}t$	**B1**
	$= 32\pi\left[\frac{1}{3}\cosh^3 t\right]_0^{\frac{3}{2}}$	**M1**
	$= \frac{32\pi}{3}\left[\left(\cosh^3\frac{3}{2}\right) - \left(\cosh^3 0\right)\right]$	**dM1**
	$= \frac{32\pi}{3}\left(\cosh^3\frac{3}{2} - 1\right)$	**A1**
		(4)
		(12 marks)

Notes:

(a)

B1: Correct derivatives.

M1: Square and adds their two derivatives (allow if coefficients not squared correctly).

M1: Applies the double argument formula for hyperbolics.

M1: Factors out the cosh *t* terms and applies Pythagorean identity for hyperbolics.

A1: Correct answer from correct work.

(b)

M1: Applies the arc length formula with the square root of their (a) and applies the double argument formula $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ to reach integrable form.

dM1: Integrates and substitutes the limits (allow bottom limit 0 implied). **Dependent on the previous method mark**.

A1: Correct answer.

Mixing up variables e.g. x and t etc loses final A mark if not recovered.

(c)

B1: States or applies the surface area formula with the square root of their (a) and y substituted, including the 2π but limits not needed for this mark.

M1: Integrates to the correct form (limits not needed for this mark). Alternatives may be possible here.

dM1: Substitutes the limits into their integral. **Dependent on the previous method mark.**

A1: Correct exact answer, accepting equivalents. Numerical equivalent 402.72…is A0.

Alt 1 for (c):

$$S_x = 2\pi\int_0^{\frac{3}{2}} y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^3 t\,\mathrm{d}t = 32\pi\int_0^{\frac{3}{2}} \sinh t + \frac{1}{4}\left(\sinh 3t - 3\sinh t\right)\mathrm{d}t$$

$$= 32\pi\left[\cosh t + \frac{1}{4}\left(\frac{1}{3}\cosh 3t - 3\cosh t\right)\right]_0^{\frac{3}{2}} = 32\pi\left(\frac{1}{4}\cosh\frac{3}{2} + \frac{1}{12}\cosh\frac{9}{2} - \frac{1}{4} - \frac{1}{12}\right)$$

$$= \frac{8\pi}{3}\left(3\cosh\frac{3}{2} + \cosh\frac{9}{2} - 4\right)$$

Alt 2 for (c)

$$S_x = 2\pi\int_0^{\frac{3}{2}} y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^3 t\,\mathrm{d}t = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^2 \mathrm{t}\ \sinh t\mathrm{d}t$$

$$= 32\pi\left[\int_0^{\frac{3}{2}} \sinh t + \int_0^{\frac{3}{2}} \cosh^2 \mathrm{t}\ \sinh t\mathrm{d}t - \int_0^{\frac{3}{2}} \sinh t\right] = 32\pi\left[\frac{1}{3}\cosh^3 t\right]_0^{\frac{3}{2}}$$

$$== \frac{32\pi}{3}\left(\cosh^3\frac{3}{2} - 1\right)$$