Question	Scheme	Marks
7(a)	$I_n = \left[\frac{1}{4}\mathrm{e}^{4x}\tan^n x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{n}{4}\mathrm{e}^{4x}\tan^{n-1}x\sec^2 x\ \mathrm{d}x$	**B1**
	$= \left[\frac{1}{4}\mathrm{e}^{4x}\tan^n x\right]_0^{\frac{\pi}{4}} - \frac{n}{4}\int_0^{\frac{\pi}{4}} \mathrm{e}^{4x}\tan^{n-1}x\left(1+\tan^2 x\right)\mathrm{d}x$	**M1**
	$= \left(\frac{1}{4}\mathrm{e}^{\pi}\right) - (0) - \frac{n}{4}\left(I_{n-2} + I_{n+1}\right)$	**M1**
	$\Rightarrow \frac{n}{4}I_{n+1} = \frac{1}{4}\mathrm{e}^{\pi} - \frac{n}{4}I_{n-1} - I_n \Rightarrow I_{n+1} = \frac{\mathrm{e}^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	**A1**
		(4)
Alt 1:	$I_{n+1} = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x}\tan^{n-1}x\tan^2 x\ \mathrm{d}x = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x}\tan^{n-1}x\left(\sec^2 x - 1\right)\mathrm{d}x$ **M1** $= \underline{\left[\mathrm{e}^{4x}\frac{\tan^n x}{n}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 4\mathrm{e}^{4x}\frac{\tan^n x}{n}\ \mathrm{d}x} - I_{n-1}$ **B1**	**B1** **M1**
	$= \left(\frac{\mathrm{e}^{\pi}}{n}\right) - (0) - \frac{4}{n}I_n - I_{n-1}$	**M1**
	$\Rightarrow I_{n+1} = \frac{\mathrm{e}^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	**A1**
		(4)
Alt 2:	$I_{n+1} = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x}\tan^{n+1}x\ \mathrm{d}x$ $= \underline{\left[\frac{1}{4}\mathrm{e}^{4x}\tan^{n+1}\mathrm{d}x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{4}\mathrm{e}^{4x}(n+1)\tan^n x\sec^2 x\ \mathrm{d}x}$	**B1**
	$= \left[\frac{1}{4}\mathrm{e}^{4x}\tan^{n+1}x\right]_0^{\frac{\pi}{4}} - \frac{n+1}{4}\int_0^{\frac{\pi}{4}} \mathrm{e}^{4x}\tan^n x\left(1+\tan^2 x\right)\mathrm{d}x$	**M1**
	$= \left(\frac{1}{4}\mathrm{e}^{\pi}\right) - (0) - \frac{n+1}{4}\left(I_n + I_{n+2}\right)$	**M1**
	$\Rightarrow I_{n+2} = \frac{\mathrm{e}^{\pi}}{n+1} - \frac{4}{n+1}I_{n+1} - I_n$ Replacing e.g. $'n = n-1' \Rightarrow I_{n+1} = \frac{\mathrm{e}^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	**A1**

Alt 3	$I_n = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x} \tan^{n-2} x \tan^2 x \, \mathrm{d}x = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x} \tan^{n-2} x \left(\sec^2 x - 1\right) \mathrm{d}x$ **M1** $= \underline{\left[\mathrm{e}^{4x} \frac{\tan^{n-1} x}{n-1}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 4\mathrm{e}^{4x} \frac{\tan^{n-1} x}{n-1} \mathrm{d}x} - I_{n-2}$ **B1**	**B1** **M1**
	$I_n = \left(\frac{\mathrm{e}^{\pi}}{n-1}\right) - (0) - \frac{4}{n-1} I_{n-1} - I_{n-2}$	**M1**
	$\Rightarrow I_{n+1} = \frac{\mathrm{e}^{\pi}}{n} - \frac{4}{n} I_n - I_{n-1}$	**A1**
(b)	$I_0 = \int_0^{\frac{\pi}{4}} \mathrm{e}^{4x} \mathrm{d}x = \left[\frac{1}{4}\mathrm{e}^{4x}\right]_0^{\frac{\pi}{4}} = \ldots$	**M1**
	$= \frac{1}{4}\mathrm{e}^{\pi} - \frac{1}{4}$	**A1**
		(2)
(c)	$I_2 = \frac{\mathrm{e}^{\pi}}{1} - \frac{"4"}{1} \times 3.7002 - \frac{1}{4}\left[\mathrm{e}^{\pi} - 1\right] \quad (= 2.8047\ldots)$	**M1**
	$I_3 = \frac{\mathrm{e}^{\pi}}{2} - \frac{"4"}{2} \times "2.8047" - 3.7002 \quad (= 2.2607\ldots)$ $I_4 = \frac{\mathrm{e}^{\pi}}{3} - \frac{"4"}{3} \times "2.2607" - "2.8047" = \ldots$	**M1**
	$= 1.8945\ldots = 1.89$	**A1**
		(3)
		(9 marks)

Notes:

(a)

B1: Correct application of integration by parts on the given integral (need not be simplified).

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the integral.

M1: Evaluates the definite part of the integral and replaces appropriately by I_{n-1} and I_{n+1} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted. Condone missing arguments etc if recovered, but if not recovered then final A0.

Alt 1:

B1: Correct application of integration by parts on a suitably adapted integral (need not be simplified). Awarded after the M1 in this approach but score as the first mark.

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the original integral. (First mark awarded in this approach but score as the second mark.)

M1: Evaluates the definite part of the integral and replaces appropriately by I_{n-1} and I_{n+1} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted.

Variations on these methods may be seen but can be scored via this scheme, but the final A will need indices correctly adjusted if they work from different index at the start of the appropriate method.

Alt 2:

B1: Correct application of integration by parts on the given integral (need not be simplified).

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the integral.

M1: Evaluates the definite part of the integral and replaces appropriately by I_n and I_{n+2} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted. If I_{n+2} is correctly found, then accept a change of variable to obtain the correct reduction formula e.g. letting $'n = n - 1'$

(b)

M1: Integrates to $k\mathrm{e}^{4x}$, $k = \pm 4$ or $\pm\frac{1}{4}$ and substitutes the limits.

A1: Correct answer.

(c)

M1: Applies the reduction formula at least once with their I_0 and the given value for I_1 and $n = 1$.

M1: Proceeds to apply twice more to reach a value for I_4. Evidence of the use of (a) must be clear.

A1: Correct answer.