8 The ellipse E has equation

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

The point $P\left(4\cos\theta, 3\sin\theta\right)$ lies on E where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation for the normal to E at P is given by

$$4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$$

(4)

The normal to E at P meets the x-axis at the point A.

(b) Show that the area of triangle OAP, where O is the origin, is $k\sin 2\theta$, where k is a rational number to be determined.

(4)

(c) Hence state the value of the maximum area for triangle OAP.

(1)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 9 marks)