Question	Scheme	Marks
9(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}\times\ldots$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\ldots\times\dfrac{1\times\sqrt{1+x^2}-x\times\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}\times 2x}{\left(\sqrt{1+x^2}\right)^2}\left(=\dfrac{1}{\left(1+x^2\right)^{\frac{3}{2}}}\right)$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}\times\dfrac{1\times\sqrt{1+x^2}-x\times\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}\times 2x}{\left(\sqrt{1+x^2}\right)^2}$	A1
	$=\dfrac{1}{1-\dfrac{x^2}{1+x^2}}\times\dfrac{\sqrt{1+x^2}-x^2\left(1+x^2\right)^{-\frac{1}{2}}}{1+x^2}=\dfrac{1+x^2-x^2}{\left(1+x^2-x^2\right)\sqrt{1+x^2}}=\dfrac{1}{\sqrt{1+x^2}}$ *	A1*
		(4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\sqrt{1+x^2}}\Rightarrow y=\int\dfrac{1}{\sqrt{1+x^2}}\,\mathrm{d}x=\operatorname{arsinh}x+c$ $\Rightarrow \operatorname{artanh}\left(\dfrac{x}{\sqrt{1+x^2}}\right)=\operatorname{arsinh}x+c$	M1
	$x=0\Rightarrow\operatorname{artanh}(0)=\operatorname{arsinh}(0)+c\Rightarrow c=0$ Therefore $\operatorname{artanh}\left(\dfrac{x}{\sqrt{1+x^2}}\right)=\operatorname{arsinh}x$	A1
		(2)
		(6 marks)

Notes:

(a)

M1: Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, with correct form for the artanh, ie the $\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}$ (and this may be all)

Need not be simplified and allow if the squaring is not fully correct (e.g. numerator or denominator squared is sufficient to imply the attempt at squaring).

M1: Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ with a correct attempt at the chain rule (or product rule) used (the artanh derivative need not be correct for this mark).

A1: A correct unsimplified expression for the derivative.

A1*: Simplifies to the give answer with suitable intermediate step and no errors seen (e.g. either of those shown in the scheme).

(b)

M1: Integrates the answer to part (a) to achieve the arsinh answer with constant included and equates to the initial function.
A1: Shows the constant of integration is zero to complete the proof with conclusion, no errors.

9(a) Alt	$\tanh y = \dfrac{x}{\sqrt{1+x^2}} \Rightarrow \operatorname{sech}^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{A\sqrt{1+x^2} - Bx\left(1+x^2\right)^{-\frac{1}{2}} \times x}{\left(\sqrt{1+x^2}\right)^2}$	**M1**
	$\Rightarrow \left(1-\tanh^2 y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(1-\dfrac{x^2}{1+x^2}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	**M1**
	$\left(1-\dfrac{x^2}{1+x^2}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{1+x^2} - x^2\left(1+x^2\right)^{-\frac{1}{2}}}{1+x^2}$	**A1**
	$\Rightarrow \left(\dfrac{1+x^2-x^2}{1+x^2}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1+x^2-x^2}{\left(1+x^2\right)\sqrt{1+x^2}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{1+x^2}}$ *	**A1***
		(4)

Notes:

(a)
M1: Takes tanh of both sides and differentiates using the product or quotient rule on the RHS, achieving the form shown.
M1: Applies $\operatorname{sech}^2 y = 1 \pm \tanh^2 y$ and replaces tanh *y* to get a derivative statement in terms of *x* only.
A1: A correct unsimplified expression in terms of *x* in the derivative.
A1*: Simplifies to the give answer with suitable intermediate step and no errors seen.