1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Determine the exact values of x for which

$$\cosh 2x - 7\sinh x = 5$$

giving your answers as natural logarithms.

(7)

(Total for Question 1 is 7 marks)

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2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Determine

(a) $\displaystyle\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\,\mathrm{d}x$, giving your answer as a multiple of π

(5)

(b) $\displaystyle\int_{-1}^{4} \frac{1}{\sqrt{4x^2 - 12x + 34}}\,\mathrm{d}x$, giving your answer in the form $p\ln\left(q + r\sqrt{2}\right)$,

where p, q and r are rational numbers.

(7)

(Total for Question 2 is 12 marks)

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3. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

The line l is a tangent to H at the point $P(4\cosh\alpha, 2\sinh\alpha)$, where α is a constant, $\alpha \neq 0$

(a) Using calculus, show that an equation for l is

$$2y\sinh\alpha - x\cosh\alpha + 4 = 0$$

(4)

The line l cuts the y-axis at the point A.

(b) Find the coordinates of A in terms of α

(2)

The point B has coordinates $(0, 10\sinh\alpha)$ and the point S is the focus of H for which $x > 0$

(c) Show that the line segment AS is perpendicular to the line segment BS.

(5)

(Total for Question 3 is 11 marks)