| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2(a)** | "...show all stages of your working. Solutions relying entirely on calculator technology are not acceptable." | | |
| | $x^2+4x+13=(x+2)^2+9$ or $(x+2)^2+3^2$ | Correctly completes the square | **B1** |
| | $\int \frac{1}{(x+2)^2+9}\,dx=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$ M1: $k\arctan(\pm ax\pm b)$ $a,b\neq 0$, A1: Correct<br>$k$ can be 1. Condone e.g., "artan" if work confirms that "arctan" was intended | | **M1 A1** |
| | $\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}(\arctan 1-\arctan 0)$ | Correct use of correct limits but arctan 0 or 0 need not be shown. Could go straight to a consistent value | **dM1** |
| | $\frac{\pi}{12}$ | Correct value | **A1** |
| | May see "mini" substitutions here (& in (b)) such as $u=x+2$: limits must change correctly | | **(5)** |
| **Example by Sub--stitution** | $x^2+4x+13=(x+2)^2+9$ or $(x+2)^2+3^2$ | Correctly completes the square | **B1** |
| | $x+2=3\tan t\Rightarrow \frac{dx}{dt}=3\sec^2 t$ $\quad x=-2,\tan t=0,\ t=0;\ x=1,\tan t=1,\ t=\frac{\pi}{4}$ | | |
| | $\int \frac{3\sec^2 t}{9\tan^2 t+9}\,dt=\frac{1}{3}\int dt=\frac{1}{3}t$ | M1: Substitutes $x+2=3\tan t$, uses $\tan^2 t+1=\sec^2 t$ and integrates to $kt$<br>A1: Correct expression. Ignore limits | **M1 A1** |
| | $\left[\frac{1}{3}t\right]_0^{\frac{\pi}{4}}=\frac{1}{3}\times\frac{\pi}{4}-\frac{1}{3}\times 0,\ =\frac{\pi}{12}$ | Either changes limits correctly and substitutes correctly or reverts to $x$ correctly and uses original limits correctly. $\frac{1}{3}\times 0$ or 0 need not be seen - could go straight to a consistent value | **dM1 A1** |
| **(b)** | $4x^2-12x+34=4\left(x-\frac{3}{2}\right)^2+25$ or $(2x-3)^2+5^2$ oe May be seen within root<br>M1: $4(x\pm p)^2\pm q$ or $(2x\pm p)^2\pm q$ oe $(p,q\neq 0)$ A1: $4\left(x-\frac{3}{2}\right)^2+25$ oe e.g., $4\left(\left(x-\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2\right)$ | | **M1 A1** |
| | $\frac{1}{2}\int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{25}{4}}}\,dx=\frac{1}{2}\operatorname{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)$ or $\frac{1}{2}\operatorname{arsinh}\left(\frac{2x-3}{5}\right)$ or e.g., $\frac{1}{2}\ln\left(x-\frac{3}{2}+\sqrt{\left(x-\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2}\right)$<br>M1: $k\operatorname{arsinh} f(x)$ or $p\ln\left(f(x)+\sqrt{(f(x))^2+q}\right)$ oe $f(x)=\pm ax\pm b$ $a,b\neq 0$ A1: Correct<br>(If log form ignore any embedded multiplying constant within the ln bracket).<br>$k$ can be 1.<br>Condone e.g., "arcsinh" if recovered. Above may be obtained via substitutions e.g., $x-\frac{3}{2}=\frac{5}{2}\sinh t\left(\rightarrow kt,=\frac{1}{2}t\right)$ or (unlikely) $x-\frac{3}{2}=\frac{5}{2}\tan t\left(\rightarrow k\left\{=\frac{1}{2}\right\}\ln(\sec t+\tan t)\right)$ | | **M1 A1** |
| | $\left[\frac{1}{2}\operatorname{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^{4}$ or $\frac{1}{2}\left[t\right]_{\operatorname{arsinh}(-1)}^{\operatorname{arsinh}1}=\frac{1}{2}(\operatorname{ar\,sinh}(1)-\operatorname{ar\,sinh}(-1))$ | Correct use of correct limits (changed correctly if necessary) | **ddM1** |
| | $=\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)-\ln\left(-1+\sqrt{2}\right)\right)$ or $=\frac{1}{2}\times 2\operatorname{arsinh}(1)=\ln\left(1+\sqrt{2}\right)$<br>Uses the correct logarithmic form of arsinh twice (or once if uses $\operatorname{arsinh}(1)-\operatorname{arsinh}(-1)=2\operatorname{arsinh}(1)$). This mark may also be gained by twice using the correct exponential form of sinh$x$ and proceeding appropriately to a numerical expression. May be slips but must reach 3TQs in $e^x$ (do not have to be written in =0 form). Then solved by usual rules but allow if consistent calculator answers | | **ddM1 Requires first 2 M marks** |
| | Score ddM1 ddM1 together for correctly using correct limits on an appropriate log form, e.g., $\left[\frac{1}{2}\ln\left(\frac{2x-3}{5}+\sqrt{\left(\frac{2x-3}{5}\right)^2+1}\right)\right]_{-1}^{4}=\ldots$ or $\left[\frac{1}{2}\ln\left(2x-3+\sqrt{(2x-3)^2+25}\right)\right]_{-1}^{4}=\ldots$ $\left\{\frac{1}{2}\left(\ln\left(5+5\sqrt{2}\right)-\ln\left(-5+5\sqrt{2}\right)\right)\right\}$ | | |
| | $=\frac{1}{2}\ln\frac{1+\sqrt{2}}{-1+\sqrt{2}}=\frac{1}{2}\ln\left(\frac{1+\sqrt{2}}{-1+\sqrt{2}}\times\frac{-1-\sqrt{2}}{-1-\sqrt{2}}\right)=\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ $\left[\text{or }\ln\left(1+\sqrt{2}\right)\right]$<br>Not just values of constants unless form seen. Must include explicit surd work but it could be minimal and is not necessary with answer $\ln\left(1+\sqrt{2}\right)$. Condone missing brackets. | | **A1**<br>**(7)**<br>**Total 12** |