Question Number	Scheme	Notes	Marks
4	$I_n = \int \sec^n x \, \mathrm{d}x \qquad n \geq 0$ If d(...) notation is used it must be resolved before the relevant marks are awarded		
4(a) **Integrates first**	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$ at the start	**M1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x \tan^2 x \, \mathrm{d}x$	dM1: Uses integration by parts to obtain $\sec^{n-2} x \tan x \pm k \int \sec^{n-2} x \tan^2 x \, \mathrm{d}x$ oe A1: Any correct expression	**dM1 A1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x(\sec^2 x - 1) \, \mathrm{d}x$	Replaces $\tan^2 x$ with $\sec^2 x - 1$ Requires 1st mark	**B1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - (n-2)\int \sec^n x \, \mathrm{d}x + (n-2)\int \sec^{n-2} x \, \mathrm{d}x$ or $I_n = \sec^{n-2} x \tan x - (n-2)I_n + (n-2)I_{n-2}$ **Depends on all previous M <u>and</u> B marks.** Obtains equation in I_n and I_{n-2} or integrals - not just the final answer		**ddM1**
	$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof. Allow e.g., $(n-2)I_{n-2} + \sec^{n-2} x \tan x = (n-1)I_n$	**A1***
			(6)
ALT **Identity first**	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$ at the start	**M1**
	$\int \sec^{n-2} x \sec^2 x \mathrm{d}x = \int \sec^{n-2} x (\tan^2 x + 1)\mathrm{d}x$ $\left\{= \int \sec^{n-2} x \mathrm{d}x + \int \tan^2 x \sec^{n-2} x \mathrm{d}x\right\}$	Replaces $\sec^2 x$ with $\tan^2 x + 1$ Requires 1st mark	**B1 (4th ePen mark)**
	$\int \tan^2 x \sec^{n-2} x \mathrm{d}x = \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)} \int \sec^n x \, \mathrm{d}x$ dM1: Uses int. by parts on $\int \tan^2 x \sec^{n-2} x \mathrm{d}x$ to obtain $p \tan x \sec^{n-2} x \pm q \int \sec^n x \, \mathrm{d}x$ oe A1: Any correct expression for $\int \tan^2 x \sec^{n-2} x \mathrm{d}x$		**dM1 A1**
	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \, \mathrm{d}x + \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)} \int \sec^n x \, \mathrm{d}x$ or $I_n = I_{n-2} + \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)} I_n$ **Depends on all previous M <u>and</u> B marks.** Obtains equation in I_n and I_{n-2} or integrals - not just the final answer.		**ddM1**
	$\Rightarrow$ e.g., $(n-2+1)I_n = (n-2)I_{n-2} + \tan x \sec^{n-2} x$ $\Rightarrow (n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof. An intermediate step is required for this approach. Allow a final answer of e.g., $(n-2)I_{n-2} + \sec^{n-2} x \tan x = (n-1)I_n$	**A1***
	For either method condone missing dx's, the odd missing variable, notational issues e.g., "$\sec x^{n-2}$" and allow recovery of missing brackets if this is before the given answer. Since this question involves trigonometric and not hyperbolic functions we will condone the occasional e.g., tanh for tan but the final answer must be as shown.		

Question Number	Scheme	Notes	Marks
4(b)	"...showing each step of your working...Solutions relying entirely on calculator technology are not acceptable." $(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}, \quad I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1} I_{n-2}$		
***I*₆ first**	$I_2 \left\{ = \left[\tan x\right]_0^{\frac{\pi}{4}} \right\} = 1$	Correct **value** for I_2 seen or implied Ignore "calculations" of I_0	**B1**
	$I_6 = \frac{1}{5}\left[\tan x \sec^4 x\right]_0^{\frac{\pi}{4}} + \frac{4}{5} I_4$ or e.g. $I_6 = \frac{1}{5}\tan\frac{\pi}{4}\sec^4\frac{\pi}{4} + \frac{4}{5} I_4$ or e.g. $I_6 = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{5} I_4$ Applies the reduction formula once. No need to see or apply limits. Fine as $5I_6 = ...$ and allow slips e.g., using an incorrect formula rearrangement e.g, $I_n = \tan x \sec^{n-2} x + (n-2) I_{n-2}$ or e.g., $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + (n-2) I_{n-2}$		**M1**
	$= \frac{1}{5}\tan x \sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x \sec^2 x + \frac{2}{3} I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}(1)$ Applies the given reduction formula again (same slips as before) and uses the limits to reach a numerical expression for I_6. If just a value is given it must be $\frac{28}{15}$		**dM1**
	$= \frac{28}{15}$	Correct value (or exact equivalent)	**A1**
			(4)
ALT *I*₄ first	$I_2 = \left\{ \left[\tan x\right]_0^{\frac{\pi}{4}} = \right\} 1$	Correct **value** for I_2 seen or implied Ignore "calculations" of I_0	**B1**
	$I_4 = \frac{1}{3}\left[\tan x \sec^2 x\right]_0^{\frac{\pi}{4}} + \frac{2}{3} I_2$ or e.g. $I_4 = \frac{1}{3}\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} + \frac{2}{3} I_2$ or e.g. $I_4 = \frac{1}{3}(1)\left(\sqrt{2}\right)^2 + \frac{2}{3} I_2 \left\{ = \frac{4}{3} \right\}$ Applies the reduction formula once. No need to see or apply limits. Fine as $3I_4 = ...$ and allow slips as main scheme		**M1**
	$I_6 = \frac{1}{5}\tan x \sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x \sec^2 x + \frac{2}{3} I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}$ Applies the reduction formula again (same slips as before) and uses the limits to reach a numerical expression for I_6. If just a value is given it must be $\frac{28}{15}$		**dM1**
	$= \frac{28}{15}$	Correct value (or exact equivalent)	**A1**
			Total 10

For either method in part (b), working must be shown and the given reduction formula must be used at least once. So do not allow e.g. *I*₆ to be attempted directly but *I*₄ can be evaluated directly without using the given reduction formula using an alternative explicit method e.g. by parts or by substitution. To score the dM1 in these cases they will need to have integrated <u>correctly</u> and reached a numerical expression for *I*₆. The B mark can be awarded for a correct value for *I*₄ or *I*₆ $\left(\frac{4}{3} \text{ or } \frac{28}{15}\right)$:

<u>Parts:</u>

$$I_4 = \int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx = \sec^2 x \tan x - 2\int \sec^2 x \tan^2 x \, dx$$

$$= \sec^2 x \tan x - 2\int \sec^2 x \left(\sec^2 x - 1\right) dx = \sec^2 x \tan x - 2\int \sec^4 x \, dx + 2\int \sec^2 x \, dx$$

$$= \sec^2 x \tan x - 2I_4 + 2\int \sec^2 x \, dx \Rightarrow 3I_4 = \sec^2 x \tan x + 2\tan x \Rightarrow I_4 = \frac{1}{3}\sec^2 x \tan x + \frac{2}{3}\tan x$$

<u>Substitution:</u>

$$I_4 = \int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx = \int \sec^2 x \left(1 + \tan^2 x\right) dx$$

$$u = \tan x \Rightarrow \int \sec^2 x \left(1 + \tan^2 x\right) dx = \int \sec^2 x \left(1 + u^2\right) \frac{du}{\sec^2 x} = \frac{u^3}{3} + u = \frac{\tan^3 x}{3} + \tan x$$