4.

$$I_n = \int \sec^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^6 x \, dx$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(Total for Question 4 is 10 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 & 0 \\ k & 2 & 0 \\ -3 & -5 & k \end{pmatrix}$$

where k is a real constant, $k \neq 0, k \neq -\frac{8}{5}$

(a) Find, in terms of k, the inverse of the matrix $\mathbf{M}$.

(5)

A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix

$$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}$$

The transformation T maps the plane Π_1 onto the plane Π_2

Given that the plane Π_2 has equation $2x - z = 4$

(b) find a Cartesian equation of the plane Π_1

(6)

(Total for Question 5 is 11 marks)