Question Number	Scheme	Notes	Marks
5.	$\mathbf{M}=\begin{pmatrix} 4 & -5 & 0 \\ k & 2 & 0 \\ -3 & -5 & k \end{pmatrix}$ Ignore labelling of matrices in (a) and allow any brackets/no brackets throughout		
(a)	$\lvert\mathbf{M}\rvert = 4(2k)+5\left(k^2\right)(+0)$	Correct determinant in any form (all 2 x 2 determinants processed). May use Sarrus e.g., $8k-\left(-5k^2\right)$	**1st B1**
	Minors : $\begin{pmatrix} 2k & k^2 & -5k+6 \\ -5k & 4k & -35 \\ 0 & 0 & 8+5k \end{pmatrix}$ or Cofactors : $\begin{pmatrix} 2k & -k^2 & 6-5k \\ 5k & 4k & 35 \\ 0 & 0 & 8+5k \end{pmatrix}$ A correct first step of minors or cofactors. May have transposed. Terms collected.		**2nd B1**
	$\mathbf{M}^{-1}=\frac{1}{5k^2+8k}\begin{pmatrix} 2k & 5k & 0 \\ -k^2 & 4k & 0 \\ 6-5k & 35 & 8+5k \end{pmatrix}$	M1: A full credible attempt at inverse: minors, cofactors, transpose and x 1/ their determinant B1: Any 2 correct rows or columns in Adj(**M**) - ignore any determinant A1: Any fully correct inverse with like terms collected	**M1** **3rd B1** **A1**
			(5)
(b)	e.g., $\mathbf{M}^{-1}=-\frac{1}{3}\begin{pmatrix} -2 & -5 & 0 \\ -1 & -4 & 0 \\ 11 & 35 & 3 \end{pmatrix}$	Any evidence of recognising $k=-1$	**M1**
	$\Pi_2 : 2x-z=4 \Rightarrow x=s, y=t, z=2s-4$	Attempts parametric form. Could be slips but must be 2 parameters	**M1**
	$-\frac{1}{3}\begin{pmatrix} -2 & -5 & 0 \\ -1 & -4 & 0 \\ 11 & 35 & 3 \end{pmatrix}\begin{pmatrix} s \\ t \\ 2s-4 \end{pmatrix}=\begin{pmatrix} ... \\ ... \\ ... \end{pmatrix}$ Completes an attempt at $\mathbf{M}^{-1}\times$ their parametric form. Allow losing the $-\frac{1}{3}$. May be achieved with separate vectors provided results brought together. Could be seen as $x=...,\ y=...,\ z=...$ Must complete processing e.g., on the $-(2s-4)\Rightarrow ...$		**ddM1**
	$-\frac{1}{3}\begin{pmatrix} -2s-5t \\ -s-4t \\ 17s+35t-12 \end{pmatrix}$ or e.g., $\begin{pmatrix} \frac{2}{3}s+\frac{5}{3}t \\ \frac{1}{3}s+\frac{4}{3}t \\ -\frac{11}{3}s-\frac{35}{3}t-2s+4 \end{pmatrix}$	Correct parametric form for Π_1 May be seen as $x=...,\ y=...,\ z=...$ Terms may not be collected	**A1**
	$3x=2s+5t,\ \ 3y=s+4t,\ \ 3z=-11s-35t-6s+12$ $\Rightarrow ... \Rightarrow 11x-5y+z=4$ dddM1: Eliminates s and t to obtain a cartesian equation of correct form ("processed" with at least 4 terms but they may be uncollected). Requires all M marks. All of x, y and z must have been functions of both s and t and there must be some (minimal) algebra. May use e.g., a, b and c for x, y and z Allow for equivalent work e.g., $\begin{pmatrix} \frac{2}{3}s+\frac{5}{3}t \\ \frac{1}{3}s+\frac{4}{3}t \\ -\frac{17}{3}s-\frac{35}{3}t+4 \end{pmatrix}\Rightarrow\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}+\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{17}{3} \end{pmatrix}s+\begin{pmatrix} \frac{5}{3} \\ \frac{4}{3} \\ -\frac{35}{3} \end{pmatrix}t\Rightarrow \mathbf{n}=\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{17}{3} \end{pmatrix}\times\begin{pmatrix} \frac{5}{3} \\ \frac{4}{3} \\ -\frac{35}{3} \end{pmatrix}=\begin{pmatrix} \frac{11}{3} \\ -\frac{5}{3} \\ \frac{1}{3} \end{pmatrix}$ **and** $\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}.\begin{pmatrix} \frac{11}{3} \\ -\frac{5}{3} \\ \frac{1}{3} \end{pmatrix}=\frac{4}{3}$ $\Rightarrow \frac{11}{3}x-\frac{5}{3}y+\frac{1}{3}z=\frac{4}{3}$ A1: Any correct equation (terms collected) and in x, y and z e.g., $2z-10y+22x-8=0$ A0 if given in e.g., x_1, y_1, z_1 or a, b & c etc.		**dddM1** **A1**
			(6)
	Some guidance for alternatives overleaf		**Total 11**

Q5(b) alternatives:

$$\mathbf{M}^{-1}=\frac{1}{5k^2+8k}\begin{pmatrix}2k & 5k & 0\\ -k^2 & 4k & 0\\ 6-5k & 35 & 8+5k\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix},\quad \mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ k & 2 & 0\\ -3 & -5 & k\end{pmatrix}=\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}$$

There are obviously a lot of possible alternatives here. The mark dependency applies as before. If the learner would score more by the main scheme then apply that but otherwise try and match the overall approach to one of the following examples.

Allow the first M mark for any recognition that $k=-1$ (which may be seen in $\mathbf{M}$ or $\mathbf{M}^{-1}$)

Score the next two M marks (M1dM1) together for a full method where enough work has been done so that a cartesian equation can be produced by a straightforward calculation or algebraic manipulation. If this is not the case, only the first mark will be available (unless using the main scheme scores more marks e.g., an acceptable attempt at the parametric form of Π_2 is also seen).

Then award the first A if everything is correct so far.

Examples:

Using $\mathbf{MP}=\mathbf{Q}$ with $P(x,y,z)$ & $Q(a,b,c)\Rightarrow a=4x-5y,\ c=-3x-5y-z$ (4 marks)

$2a-c=4\Rightarrow 8x-10y+3x+5y+z=4$ (dddM1) $\Rightarrow 11x-5y+z=4$ (A1)

May work with $P(a,b,c)$ & $Q(x,y,z)$ and swap letters at the end

OR $\Pi_2: 2x-z=4$ so 3 points on Π_2 are e.g., $Q_1(2,\,0,\,0)$, $Q_2(3,\,0,\,2)$ & $Q_3(2,\,1,\,0)$

You may have to check to ensure these points are not collinear e.g., the vectors $\mathbf{Q_1Q_2}$ and $\mathbf{Q_1Q_3}$ are not multiples of each other otherwise this method is not viable. This will also be apparent if the three y-coordinates are all zero.

Could apply inverse to these points i.e., $\mathbf{P}=\mathbf{M}^{-1}\mathbf{Q}$ or could solve $\mathbf{MP}=\mathbf{Q}$

$\Rightarrow P_1\left(\frac{4}{3},\,\frac{2}{3},\,-\frac{22}{3}\right)$, $P_2(2,\,1,\,-13)$ & $P_3(3,\,2,\,-19)$

$$P_2-P_1\Rightarrow\frac{1}{3}\begin{pmatrix}2\\1\\-17\end{pmatrix},\quad P_3-P_1\Rightarrow\frac{1}{3}\begin{pmatrix}5\\4\\-35\end{pmatrix}\Rightarrow\text{normal by vector product}:\begin{pmatrix}11\\-5\\1\end{pmatrix}\text{ (first four marks scored)}$$

$\Pi_1: 11x-5y+z=11\left(\frac{4}{3}\right)-5\left(\frac{2}{3}\right)-\frac{22}{3}=\ldots 4$ (dddM1A1)

OR A point on Π_2 is $(2,\,0,\,0)$ and 2 vectors (must have different directions) in Π_2 are $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and $\begin{pmatrix}0\\1\\0\end{pmatrix}$

Point becomes $\left(\frac{4}{3},\,\frac{2}{3},\,-\frac{22}{3}\right)$ and vectors become $\begin{pmatrix}\frac{2}{3}\\\frac{1}{3}\\-\frac{17}{3}\end{pmatrix}$ and $\begin{pmatrix}\frac{5}{3}\\\frac{4}{3}\\-\frac{35}{3}\end{pmatrix}\Rightarrow\text{normal}:\begin{pmatrix}11\\-5\\1\end{pmatrix}$ as before

OR: The following approach using row vectors for the normal using **M** is is also valid:

$$\Pi_2: 2x-z=4 \text{ has normal } \begin{pmatrix}2\\0\\-1\end{pmatrix} \text{ and so } \Pi_1 \text{ normal is } \begin{pmatrix}2 & 0 & -1\end{pmatrix}\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}=\begin{pmatrix}11 & -5 & 1\end{pmatrix}$$

and e.g., $Q_1(2,\,0,\,0)\Rightarrow P_1\left(\frac{4}{3},\,\frac{2}{3},\,-\frac{22}{3}\right)$ (4 marks)

$\Pi_1: 11x-5y+z=11\left(\frac{4}{3}\right)-5\left(\frac{2}{3}\right)-\frac{22}{3}=\ldots 4$ (dddM1A1)

Could obtain the normal using the transpose of **M**: $\begin{pmatrix}4 & -1 & -3\\ -5 & 2 & -5\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}2\\0\\-1\end{pmatrix}=\begin{pmatrix}11\\-5\\1\end{pmatrix}$

But note that this approach needs a point on Π_1 - i.e., there must be work to achieve the "4" (which just happens to be common to both planes).