Question Number	Scheme	Notes	Marks
6.	$x = \cosh t + t, \quad y = \cosh t - t$		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \sinh t + 1, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \sinh t - 1$	Correct derivatives. Move to guidance overleaf at the point at which they move to exponentials if applicable and may see a combination of approaches. Could be implied	**B1**
	$(\sinh t + 1)^2 + (\sinh t - 1)^2$ $\{= \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1\}$ $\{= 2\sinh^2 t + 2\}$	Uses $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$ Correct for their derivatives. No need to expand but if it is only seen expanded the form must be correct (sign/coefficient errors only)	**M1**
	$\left\{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 =\right\} 2(1 + \sinh^2 t)$ or $= 2 + 2\sinh^2 t = 2\cosh^2 t$* Uses either intermediate step shown to complete the proof with no errors. A0 if e.g., sin accidentally used for sinh, x used for t, but condone $\sinh t^2$ used instead of $\sinh^2 t$ or $(\sinh t)^2$ & condone the occasional missing variable. "Meet in the middle" approaches need a conclusion.		**A1***
			(3)
(b)	$\{S = 2\pi\int y\,\mathrm{d}s =\} \; 2\pi\int(\cosh t - t)\sqrt{2\cosh^2 t}\,\{\mathrm{d}t\}$ Uses $S = 2\pi\int y\,\mathrm{d}s$ with the given y (not x) & the result from (a). Allow if x used anywhere for t, condone missing brackets and limits and dt not required		**M1**
	$S = 2\pi\sqrt{2}\int_0^{\ln 3}(\cosh^2 t - t\cosh t)\,\mathrm{d}t$* Reaches given answer with no errors (condone missing brackets) with "$S =$" seen at some point in this part. Limits can appear at end. Allow e.g., $S = 2\sqrt{2}\,\pi\int_0^{\ln 3} -t\cosh t + \cosh^2 t\,\mathrm{d}t$*		**A1***
			(2)
(c)	"Solutions relying entirely on calculator technology are not acceptable."		
	$\int\cosh^2 t\,\mathrm{d}t = \int\left(\frac{1}{2} + \frac{1}{2}\cosh 2t\right)\mathrm{d}t$	Uses/states $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ Does not need to be seen as integrand	**M1**
	$\int t\cosh t\,\mathrm{d}t = t\sinh t - \int\sinh t\,\mathrm{d}t$ M1: Applies integration by parts the right way round on $kt\cosh t$ and obtains the correct form (could have +). If only seen fully integrated must be $kt\cosh t \to k(t\sinh t - \cosh t)$ A1: Correct expression. Could be implied		**M1** **A1**
	$\Rightarrow 2\sqrt{2}\pi\int(\cosh^2 t - t\cosh t)\,\mathrm{d}t = 2\sqrt{2}\pi\left[\frac{1}{2}t + \frac{1}{4}\sinh 2t - t\sinh t + \cosh t\right]$ oe A1: Correct form (sign/coeff. slips only inc. e.g. missing/wrong $2\sqrt{2}\pi$) A1: All correct. May use e.g., $\sinh 2t \to 2\sinh t\cosh t$ and may be unsimplified (**Note that both M marks required for these marks**). Must be combined but this could be implied later.		**A1** **A1**
	$= 2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3 + \frac{10}{9} - \frac{4}{3}\ln 3 + \frac{5}{3}\right) - (1)\right\}$	Uses limits 0 and ln3 with an expression of the correct form. Hyperbolics credibly evaluated (and no "e"s). Allow any recognisable attempt at substitution here but look for subtraction of $\mathrm{f}(0) \neq 0$	**ddM1**
	$\{S = 2\sqrt{2}\,\pi(\frac{16}{9} - \frac{5}{6}\ln 3)\}$ $\Rightarrow \{S =\}\frac{\pi\sqrt{2}}{9}(32 - 15\ln 3)$	This answer but allow e.g., $\sqrt{2}\pi\left(\frac{32 - 15\ln 3}{9}\right)$ Not just values for the constants unless form seen	**A1**
	Condone confusion with "t" and "x"		**(7)**
	See overleaf for examples using exponential definitions and some general guidance for other approaches to the integration in (c).		**Total 12**

Q6(a): Using exponential definitions:

$$x=\cosh t+t=\frac{\mathrm{e}^{t}+\mathrm{e}^{-t}}{2}+t,\quad y=\cosh t-t=\frac{\mathrm{e}^{t}+\mathrm{e}^{-t}}{2}-t$$

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{2}+1,\quad \frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{2}-1 \text{ (B1)}$$

$$\left(\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{2}+1\right)^{2}+\left(\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{2}-1\right)^{2} \text{ (M1)}$$

$$=\frac{1}{2}\left(\mathrm{e}^{t}-\mathrm{e}^{-t}\right)^{2}+2 \text{ or e.g., } =\frac{1}{2}\left(\mathrm{e}^{2t}+2+\mathrm{e}^{-2t}\right)$$

$$=2\left(\frac{\mathrm{e}^{t}+\mathrm{e}^{-t}}{2}\right)^{2}=2\cosh^{2}t*$$

(A1*: Must see $k\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}$ **and** at least one additional line after substitution)

Q6(c) example using exponential definitions:

$$\int\cosh^{2}t\,\mathrm{d}t=\int\left(\frac{\mathrm{e}^{t}+\mathrm{e}^{-t}}{2}\right)^{2}\mathrm{d}t=\frac{1}{4}\int\left(\mathrm{e}^{2t}+2+\mathrm{e}^{-2t}\right)\mathrm{d}t \text{ (M1: integrable form - sign errors only)}$$

$$\int t\cosh t\,\mathrm{d}t=\frac{1}{2}\int\left(t\mathrm{e}^{t}+t\mathrm{e}^{-t}\right)\mathrm{d}t=\frac{1}{2}\left(t\mathrm{e}^{t}-\int\mathrm{e}^{t}\,\mathrm{d}t-t\mathrm{e}^{-t}+\int\mathrm{e}^{-t}\,\mathrm{d}t\right)$$

(M1: applies parts, twice if necessary, correct form A1: All correct for the integral of $t\cosh t$)

$$\Rightarrow\left(2\sqrt{2}\pi\right)\left(\left(\frac{1}{8}\mathrm{e}^{2t}+\frac{1}{2}t-\frac{1}{8}\mathrm{e}^{-2t}\right)-\left(\frac{1}{2}t\mathrm{e}^{t}-\frac{1}{2}\mathrm{e}^{t}-\frac{1}{2}t\mathrm{e}^{-t}-\frac{1}{2}\mathrm{e}^{-t}\right)\right)$$

A1: Correct form (sign/coefficient slips only) A1: All correct. May be unsimplified

$$\Rightarrow 2\sqrt{2}\pi\left[\frac{1}{8}\mathrm{e}^{2t}+\frac{1}{2}t-\frac{1}{8}\mathrm{e}^{-2t}-\frac{1}{2}t\mathrm{e}^{t}+\frac{1}{2}\mathrm{e}^{t}+\frac{1}{2}t\mathrm{e}^{-t}+\frac{1}{2}\mathrm{e}^{-t}\right]_{0}^{\ln 3}$$

$$=2\sqrt{2}\pi\left(\left(\frac{9}{8}+\frac{1}{2}\ln 3-\frac{1}{72}-\frac{3}{2}\ln 3+\frac{3}{2}+\frac{1}{6}\ln 3+\frac{1}{6}\right)-\left(\frac{1}{8}+0-\frac{1}{8}-0+\frac{1}{2}+0+\frac{1}{2}\right)\right)$$

ddM1: Uses limits 0 and ln3. No exponentials.
Allow any credible attempt at substitution here but look for subtraction of $\mathrm{f}(0)\neq 0$

$$\left\{S=2\sqrt{2}\,\pi\left(\frac{16}{9}-\frac{5}{6}\ln 3\right)=\right\}\ \frac{\pi\sqrt{2}}{9}\left(32-15\ln 3\right)\ \text{A1 as before}$$

Q6(c): For other potentially creditworthy attempts at the integration that work on the two terms separately, the following guidance applies:

M1: Reaches an integrable form for $\cosh^{2}t$ (sign/coefficient errors only)

M1: Correct form for $\int t\cosh t\,\mathrm{d}t$ A1: Fully correct integration of $\int t\cosh t\,\mathrm{d}t$

A1: Correct form for S A1: Fully correct S

ddM1A1: As main scheme

Using parts on $\cosh t\left(\cosh t-t\right)$:

$$=\sinh t\left(\cosh t-t\right)-\int\left(\sinh^{2}t-\sinh t\right)\mathrm{d}t \text{ (2nd M1: correct form, A1: fully correct)}$$

$$=\sinh t\left(\cosh t-t\right)-\int\left(-\tfrac{1}{2}+\cosh 2t-\sinh t\right)\mathrm{d}t \text{ (1st M1: } \sinh^{2}t=\pm\tfrac{1}{2}\pm\tfrac{1}{2}\cosh 2t\text{)}$$

$$\Rightarrow 2\sqrt{2}\,\pi\left[\sinh t\cosh t-t\sinh t+\tfrac{1}{2}t-\tfrac{1}{4}\sinh 2t+\cosh t\right] \text{ (A1: correct form, A1: fully correct)}$$

Then ddM1A1 as main scheme