6. The curve C has parametric equations

$$x = \cosh t + t \qquad y = \cosh t - t \qquad 0 \leqslant t \leqslant \ln 3$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 2\cosh^2 t$$

(3)

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(b) Show that

$$S = 2\pi\sqrt{2}\int_0^{\ln 3}\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$$

(2)

(c) Hence find the value of S, giving your answer in the form

$$\frac{\pi\sqrt{2}}{9}\left(a + b\ln 3\right)$$

where a and b are constants to be determined.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(7)

(Total for Question 6 is 12 marks)

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7. With respect to a fixed origin O, the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.

(a) Find the volume of the tetrahedron $ABCD$.

(4)

The plane Π contains the points A, B, and C.

(b) Find a Cartesian equation of Π.

(4)

The point T lies on the plane Π.

The line DT is perpendicular to Π.

(c) Find the exact coordinates of the point T.

(4)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS