Question Number	Scheme	Notes	Marks
4	$x=4\cos^3\theta \quad y=4\sin^3\theta \quad 0 \square \theta ,, k \quad 0<k<\dfrac{\pi}{2}$		
(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-12\cos^2\theta\sin\theta$ $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=12\sin^2\theta\cos\theta$	M1: Both derivatives of the correct form. Look for $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=p\cos^2\theta\sin\theta$ and $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=q\sin^2\theta\cos\theta$ o.e. A1: Both correct derivatives. Be aware of alternative ways to differentiate, e.g. first writing $x=4\left(\cos^2\theta\right)\cos\theta$ and $y=4\left(\sin^2\theta\right)\sin\theta$. M1A1 can be given for correct derivatives, but it is unlikely further progress will be made.	M1 A1
	$S=2\pi\int_{\{0\}}^{\{k\}} y\sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\ \{\mathrm{d}\theta\}$ $=2\pi\int_{\{0\}}^{\{k\}} 4\sin^3\theta\sqrt{144\cos^4\theta\sin^2\theta+144\sin^4\cos^2\theta}\ \{\mathrm{d}\theta\}$	Uses a correct formula with their derivatives	M1
	$=96\pi\int_{\{0\}}^{\{k\}}\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}\ \{\mathrm{d}\theta\}$ $=96\pi\int_{\{0\}}^{\{k\}}\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta}\ \{\mathrm{d}\theta\}$ $\left\{=96\pi\int_{\{0\}}^{\{k\}}\sin^3\theta\cos\theta\sin\theta\,\mathrm{d}\theta\right\}$	Uses $\cos^2\theta+\sin^2\theta=1$ oe at some point in the working to process the integrand (may be implied: see the note for the A* mark below)	M1
	$=96\pi\int_0^k\sin^4\theta\cos\theta\,\mathrm{d}\theta$ *	Obtains the given answer with no errors. Must be at least one intermediate step following $96\pi\int_0^k\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}\,\mathrm{d}\theta$, possibly in correct side working. Proceeding from this expression directly to the given answer is M1 A0*. Condone the limits and/or $\mathrm{d}\theta$ appearing for the first time in the final line. Do not condone invisible brackets at any stage, even if recovered, but do condone a missing trailing bracket.	A1*
			(5)

(b)	$S=\frac{96\pi}{5}\left[\sin^5\theta\right]_0^k$	Integrates to obtain $p\sin^5\theta$	M1
	$\frac{96\pi}{5}\left(\sin^5 k\right)=\frac{27\sqrt{3}}{5}\pi$ $\Rightarrow \sin^5 k=\frac{9\sqrt{3}}{32}$ $\Rightarrow \sin k=...$	Substitutes k, sets equal to $\frac{27\sqrt{3}}{5}\pi$ and solves for sin k. **Note:** - Condone no working with the lower limit - Some candidates do not write down a value for $\sin k$, but go from $\sin^5 k$ to k. This scores M1 if their k is consistent with their $\sin^5 k$ - $k=\frac{\sqrt{3}}{2}$ o.e. is M0, unless there is recovery - This mark is not dependent on the first M1	M1
	$\sin k=\frac{\sqrt{3}}{2}$ o.e. or 0.866... $\Rightarrow k=\frac{\pi}{3}$ or awrt 1.05	Correct k	A1
			(3)
(c)	$s=\int_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta=...$ $=12\int_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}\cos\theta\sin\theta\,\mathrm{d}\theta$	Attempts to use a correct formula with their derivatives. If no formula is quoted, condone an incorrect or missing multiplying constant, which should be 12. Ignore the limits for this mark.	M1
	$=-6\left[\cos^2\theta\right]_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}$ or $6\left[\sin^2\theta\right]_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}$ or $=6\int_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}\sin 2\theta\,\mathrm{d}\theta=-3\left[\cos 2\theta\right]_{\{0\}}^{\left\{\frac{\pi}{3}\right\}}$	Integrates to $...\cos^2\theta$ or $...\sin^2\theta$ or $...\cos 2\theta$	M1
	$=-6\left(\frac{1}{4}-1\right)$ or $6\left(\frac{3}{4}-0\right)$ or $-3\left(-\frac{1}{2}-1\right)=\frac{9}{2}$	Correct exact value $\frac{9}{2}, 4.5$ or $4\frac{1}{2}$	A1
			(3)
			Total 11