4.

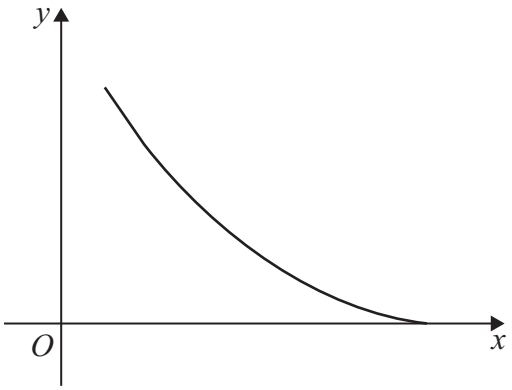


Figure 1

Figure 1 shows the curve with parametric equations

$$x = 4\cos^3\theta \qquad y = 4\sin^3\theta \qquad 0 \leqslant \theta \leqslant k$$

where k is a positive constant and $k < \frac{\pi}{2}$

The curve is rotated through 2π radians about the x-axis.

The area of the curved surface generated is given by S.

(a) Show that

$$S = 96\pi \int_0^k \sin^4\theta \cos\theta \, \mathrm{d}\theta$$

(5)

Given that $S = \frac{27\sqrt{3}}{5}\pi$

(b) use algebraic integration to determine the value of k.

(3)

(c) Hence use algebraic integration to determine the length of the curve.

(3)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 11 marks)