Question Number	Scheme	Notes	Marks
7	$P(6, 1, -2)$ $\quad \mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $\quad \mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ $\quad Q(0, 9, 32)$		
(a)	$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$	Attempts vector product with two components correct.	
	Alternatively, if the normal vector is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, then: $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \Rightarrow 2a + 3b - c = 0$; and $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \Rightarrow 4a - b + 2c$ With solutions, e.g. $a = 5, b = -8, c = -14$	Alternatively, attempts to find the three components a, b and c of the normal vector using scalar products with the two vectors in the plane, forming two equations in a, b and c and solving.	M1
	$\mathbf{r} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$	Applies **r.n = a.n** If $\begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$ is not shown, then it can be implied by 50. If no working and an incorrect value, then M0.	M1
	$5x - 8y - 14z = 50$	Any correct Cartesian equation	A1
Alt (Using sim equations)	$x = 6 + 2s + 4t \quad (1)$ $y = 1 + 3s - t \quad (2)$ $z = -2 - s + 2t \quad (3)$ e.g. (1) and (3) $\Rightarrow x + 2z = 2 + 8t \quad (4)$	Forms three equations in x, y, z, s and t, and eliminates one of s or t	M1
	e.g. (2) and (3) $\Rightarrow y + 3z = -5 + 5t \quad (5)$ e.g. (4) and (5) $\Rightarrow 5x - 8y - 14z = 50$	Eliminates both s and t Correct equation of plane	M1 A1
			(3)

(b)	Line from Q to Π in the direction of **n**: $\{\mathbf{r} = \}\begin{pmatrix} 5t \\ 9-8t \\ 32-14t \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 9 \\ 32 \end{pmatrix} + \begin{pmatrix} 5t \\ -8t \\ -14t \end{pmatrix}$	Attempts parametric form with two components correct for their normal	M1
	$5(5t) - 8(9-8t) - 14(32-14t) = 50$ $\Rightarrow 285t - 520 = 50 \Rightarrow t = 2$ **Alt 1**: Use distance of Q from plane: Distance from $Q(0, 9, 32)$ to plane is $\frac{\lvert 0(5) + 9(-8) + 32(-14) - 50 \rvert}{\sqrt{5^2 + 8^2 + 14^2}} = \frac{570}{\sqrt{285}}$ Distance from $R(5t, 9-8t, 32-14t)$ to plane is $\frac{\lvert 5t(5) + (9-8t)(-8) + (32-14t)(-14) - 50 \rvert}{\sqrt{5^2 + 8^2 + 14^2}} = \frac{\lvert 285t - 570 \rvert}{\sqrt{285}}$ $\frac{\lvert 285t - 570 \rvert}{\sqrt{285}} = \frac{570}{\sqrt{285}} \Rightarrow t = 0, 4$, so $t = 4$ at R **Alt 2**: Construct a parallel plane through Q: $\mathbf{r} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ 32 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix} = -520$, or $\mathbf{r} \cdot \hat{\mathbf{n}} = \frac{-520}{\sqrt{285}}$. Equation of Π is $\mathbf{r} \cdot \hat{\mathbf{n}} = \frac{50}{\sqrt{285}}$. Distance between planes is $\frac{50}{\sqrt{285}} - \frac{-520}{\sqrt{285}} = \frac{570}{\sqrt{285}}$. Then consider R as above.	Substitutes their parametric form into their plane equation and solves for their parameter. **Alt 1:** Finds the distance from Q to plane; finds distance from $R(5t, 9-8t, 32-14t)$ to plane; equates and solves to find t **Alt 2:** Constructs a parallel plane through Q and calculates distance between the two planes.	M1
	$t = 4 \Rightarrow R(20, -23, -24)$ **or** $t = 2 \Rightarrow X(10, -7, 4)$ on the plane. Then $\frac{Q+R}{2} = X \Rightarrow R(20, -23, -24)$	M1: Substitutes the appropriate value of t into their equation of line to obtain a point; **or** finds the point X on the plane where the line meets the plane and uses a correct midpoint calculation to find R. A1: Correct point. Condone vector form.	M1 A1
			(4)

(c) (Using vector product)	$\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 8 \\ 34 \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 14 \\ -24 \\ -22 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 20 \\ -32 \\ -56 \end{pmatrix}$ e.g. $\overrightarrow{PQ} \times \overrightarrow{PR} = ...$	Uses subtraction to obtain two relevant vectors and attempts a vector product. You do not need to check their answer.	M1
	$\text{Area} = \frac{1}{2}\left\|\begin{pmatrix} 640 \\ 344 \\ 32 \end{pmatrix}\right\| = \frac{1}{2}\sqrt{640^2 + 344^2 + 32^2} = ...$	Obtains a value from a correct method for the area of the triangle	M1
	$= \frac{1}{2}\sqrt{528960} = 4\sqrt{8265}$	Correct exact value in required form	A1
Alt (Using right-angled triangles)	e.g. QR intersects Π at $X(10, -7, 4)$ e.g., $\overrightarrow{PX} = \begin{pmatrix} 4 \\ -8 \\ 6 \end{pmatrix}$, $\overrightarrow{QX} = \begin{pmatrix} 10 \\ -16 \\ -28 \end{pmatrix}$ $PX = \sqrt{4^2 + 8^2 + 6^2}$ $\quad$ $QX = \sqrt{10^2 + 16^2 + 28^2}$	Attempts base and perpendicular height of a relevant triangle	M1
	$\text{Area} = 2 \times \text{Area}\,\Delta PQX = 2 \times \frac{1}{2} \times 2\sqrt{29} \times 2\sqrt{285} = ...$	Obtains a value from a correct method for the area of the triangle	M1
	$= 4\sqrt{8265}$	Correct exact value in required form	A1
Alt (Using area of triangle formula)	$\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 8 \\ 34 \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 14 \\ -24 \\ -22 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 20 \\ -32 \\ -56 \end{pmatrix}$ e.g. $\lvert PQ\rvert = 2\sqrt{314}$, $\lvert PR\rvert = 2\sqrt{314}$, $\lvert QR\rvert = 4\sqrt{285}$ By cosine rule, $\cos QPR = -\frac{128}{157} \Rightarrow \sin QPR = 0.579...$	Uses a correct method to find one of the angles of triangle PQR or its sine.	M1
	$\text{Area} = \frac{1}{2}\left(2\sqrt{314}\right)\left(2\sqrt{314}\right)(0.579...)$	Uses a correct area of triangle formula	M1
	$= 4\sqrt{8265}$	Correct exact value in required form. Note that methods involving an angle are unlikely to result in an exact value for the area, so probably score A0	A1
	Note: There are many variations on the above. If the right-angled triangle PQX is considered, angles may be calculated using SOHCAHTOA and side lengths using Pythagoras' Theorem.		
			(3)
			Total 10