Question Number	Scheme	Notes	Marks
8	$I_n = \int_0^{\frac{1}{2}} x^n e^{-2x}\,dx \quad J_n = \int_0^{\frac{1}{2}} x^n e^{2x}\,dx \quad n \geqslant 0$		
(a)	$I_n = -\frac{1}{2}\left[x^n e^{-2x}\right]_0^{\frac{1}{2}} + \frac{n}{2}\int_0^{\frac{1}{2}} x^{n-1} e^{-2x}\,dx$ o.e.	M1: Applies integration by parts and obtains an expression of the correct form, allowing for slips with the numerical coefficients. Limits and dx are not required for this mark. A1: A correct expression. Condone missing dx. This mark requires correct limits, but condone invisible limits on either term if implied by subsequent work.	M1 A1
	$= -\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)\left(\frac{1}{e}\right) + \frac{n}{2}I_{n-1}$ $= \frac{n}{2}I_{n-1} - \frac{1}{2^{n+1}e}$ *	Obtains given answer with an intermediate step and no errors. Following integration by parts, the limits and dx must be seen on the integral term at some point in the working for this mark.	A1*
			(3)
(b)	$\int_0^{\frac{1}{2}} x^n \sinh 2x\,dx = \int_0^{\frac{1}{2}} x^n\left(\frac{e^{2x} - e^{-2x}}{2}\right)dx = \frac{J_n - I_n}{2}$ *	Correct proof. If working from left to right, it is not necessary to see the starting expression $\int_0^{\frac{1}{2}} x^n \sinh 2x\,dx$, but correct limits must be seen at least once. If working from right to left, dx and the correct limits must be seen in the final answer.	B1*
			(1)

(c)	**Note:** For the first two method marks, condone slips. The marks can be awarded for an attempt, using the reduction formulae, to find formula(e) allowing them to find J_3 from J_0 and I_3 from I_0.		
	$J_3 = \frac{e}{16} - \frac{3}{2}\left(\frac{e}{8} - \left(\frac{e}{4} - \frac{1}{2}J_0\right)\right)$ $I_3 = -\frac{1}{16e} + \frac{3}{2}\left(-\frac{1}{8e} - \frac{1}{4e} + \frac{1}{2}I_0\right)$	M1: Uses reduction formulae to attempt to obtain J_3 in terms of J_0 **or** I_3 in terms of I_0 M1: Uses reduction formulae to attempt to obtain J_3 in terms of J_0 **and** I_3 in terms of I_0	M1 M1
	Alt for M1 M1 $J_3 = \frac{1}{16}e - \frac{3}{2}J_2$ $J_2 = \frac{1}{8}e - J_1$ $J_1 = \frac{1}{4}e - \frac{1}{2}J_0$ $I_3 = -\frac{1}{16}e^{-1} + \frac{3}{2}I_2$ $I_2 = -\frac{1}{8}e^{-1} + I_1$ $I_1 = -\frac{1}{4}e^{-1} + \frac{1}{2}I_0$	M1: Attempts to find J_3 in terms of J_2, J_2 in terms of J_1 and J_1 in terms of J_0 **or** I_3 in terms of I_2, I_2 in terms of I_1 and I_1 in terms of I_0 M1: Attempts to find J_3 in terms of J_2, J_2 in terms of J_1 and J_1 in terms of J_0 **and** I_3 in terms of I_2, I_2 in terms of I_1 and I_1 in terms of I_0	M1 M1 (Alt)
	$J_0 = \int_0^{\frac{1}{2}} e^{2x}\,dx = \frac{1}{2}\left[e^{2x}\right]_0^{\frac{1}{2}} = \frac{1}{2}e - \frac{1}{2}$ $I_0 = \int_0^{\frac{1}{2}} e^{-2x}\,dx = -\frac{1}{2}\left[e^{-2x}\right]_0^{\frac{1}{2}} = -\frac{1}{2e} + \frac{1}{2}$	For either J_0 or I_0 correct in terms of e.	B1
	For reference $J_1 = \frac{1}{4}$ $J_2 = \frac{e}{8} - \frac{1}{4}$ $J_3 = \frac{3}{8} - \frac{e}{8}$ $I_1 = \frac{1}{4} - \frac{1}{2e}$ $I_2 = \frac{1}{4} - \frac{3}{8e}$ $I_3 = \frac{3}{8} - \frac{1}{e}$		
	$\int_0^{\frac{1}{2}} x^3 \sinh 2x\,dx = \frac{1}{2}\left(\frac{e}{8} - \left(-\frac{1}{e}\right)\right)$	Attempts $\frac{J_3 - I_3}{2}$. J_3 and I_3 must be in terms of e and/or J_0 or I_0 respectively. **Requires both previous M marks.**	**dd**M1
	$= \frac{1}{2e} - \frac{e}{16} = \frac{1}{16}\left(\frac{8}{e} - e\right)$	Correct answer in the required form	A1

(c) Alt for M1 M1 B1	Some candidates are finding J_3 and I_3 in terms of J_1 and I_1, then finding J_1 and I_1 using integration by parts.		
	$J_3 = \frac{e}{16} - \frac{3}{2}\left(\frac{e}{8} - J_1\right)$ $I_3 = -\frac{1}{16e} + \frac{3}{2}\left(I_1 - \frac{1}{8e}\right)$ **or** $J_3 = \frac{1}{16}e - \frac{3}{2}J_2,\ J_2 = \frac{1}{8}e - J_1$ $I_3 = -\frac{1}{16}e^{-1} + \frac{3}{2}I_2,\ I_2 = -\frac{1}{8}e^{-1} + I_1$	M1: Uses reduction formulae to obtain J_3 in terms of J_1 **or** I_3 in terms of I_1 M1: Uses reduction formulae to obtain both J_3 in terms of J_1 **and** I_3 in terms of I_1	M1 M1
	$J_1 = \int_0^{\frac{1}{2}} xe^{2x}dx = \frac{1}{2}\left[xe^{2x}\right]_0^{\frac{1}{2}} - \frac{1}{2}\int_0^{\frac{1}{2}} e^{2x}dx = \frac{1}{4}e - \frac{1}{4}(e-1) = \frac{1}{4}$ $I_1 = \int_0^{\frac{1}{2}} xe^{-2x}dx = -\frac{1}{2}\left[xe^{-2x}\right]_0^{\frac{1}{2}} + \frac{1}{2}\int_0^{\frac{1}{2}} e^{-2x}dx = -\frac{1}{4e} - \frac{1}{4}\left(\frac{1}{e} - 1\right) = \frac{1}{4} - \frac{1}{2e}$ For either J_1 or I_1 correct in terms of e. If working is shown, it must be a correct application of integration by parts. $\left\{\Rightarrow J_3 = \frac{3}{8} - \frac{e}{8},\ I_3 = \frac{3}{8} - \frac{1}{e}\right\}$		B1
	Then ddM1 A1 as above		
			(5)
			Total 9
			PAPER TOTAL: 75