8.

$$I_n = \int_0^{\frac{1}{2}} x^n \mathrm{e}^{-2x} \mathrm{d}x \qquad n \geqslant 0$$

(a) Show that, for $n > 0$

$$I_n = \frac{n}{2} I_{n-1} - \frac{1}{2^{n+1}\mathrm{e}}$$

(3)

$$J_n = \int_0^{\frac{1}{2}} x^n \mathrm{e}^{2x} \mathrm{d}x \qquad n \geqslant 0$$

(b) Show that, for $n \geqslant 0$

$$\int_0^{\frac{1}{2}} x^n \sinh 2x \, \mathrm{d}x = \frac{J_n - I_n}{2}$$

(1)

Hence given that, for $n > 0$

$$J_n = \frac{\mathrm{e}}{2^{n+1}} - \frac{n}{2} J_{n-1}$$

(c) show that

$$\int_0^{\frac{1}{2}} x^3 \sinh 2x \, \mathrm{d}x = \frac{1}{16}\left(\frac{k}{\mathrm{e}} - \mathrm{e}\right)$$

where k is an integer to be determined.

(5)

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

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