Question Number	Scheme	Notes	Marks
Note: Attempts that don't integrate, or that use the given standard integral in the formula book will generally score max B1M0A0			
2(a)	$x = \tanh u \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = \operatorname{sech}^2 u$	Sets $x = \tanh u$ and differentiates correctly. Condone different or interchanged variables as long as the differentiation is correct	**B1**
	$\displaystyle\int \frac{1}{1-x^2}\,\mathrm{d}x = \int \frac{\operatorname{sech}^2 u}{1-\tanh^2 u}\,\mathrm{d}u = \ldots$	A full substitution, correct for their x and $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ and integrates. Allow $\operatorname{sech}^2 u = \pm 1 \pm \tanh^2 u$ If there are clear sign errors but the method is otherwise correct, allow the M mark but deduct the A1* mark	**M1**
	$= \displaystyle\int 1\,\mathrm{d}u = u + c = \operatorname{artanh} x + c$	Correct proof with one intermediate step. Penalise notational errors e.g. missing 'h' or constant 'c' etc but allow equivalent notation for $\operatorname{ar\,tanh} x$ such as $\tanh^{-1} x$ but not $\operatorname{arc\,tanh} x$	**A1***
			(3)
Alt	$\dfrac{1}{1-x^2} = \dfrac{1}{2}\left(\dfrac{1}{1+x} + \dfrac{1}{1-x}\right)$	Correct partial fractions	**B1**
	$\displaystyle\int \frac{1}{1-x^2}\,\mathrm{d}x = \frac{1}{2}\int\left(\frac{1}{1+x} + \frac{1}{1-x}\right)\mathrm{d}x = \frac{1}{2}\left(\ln(1+x) - \ln(1-x)\right) \quad [+c]$ Correctly integrates partial fractions of the correct form		**M1**
	$= \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right) + c = \operatorname{artanh} x + c$	Correct proof with one intermediate step. Condone the omission of brackets. Penalise notational errors e.g. missing 'h' or constant 'c' etc but allow equivalent notation for $\operatorname{ar\,tanh} x$ such as $\tanh^{-1} x$ but not $\operatorname{arc\,tanh} x$	**A1***
			(3)
SC	$y = \operatorname{ar\,tanh} x \Rightarrow x = \tanh y \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \operatorname{sech}^2 y$ $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \operatorname{sech}^2 y = 1 - \tanh^2 y \Rightarrow y = \displaystyle\int \frac{1}{1-\tanh^2 y}\,\mathrm{d}x$ $\Rightarrow y = \displaystyle\int \frac{1}{1-x^2}\,\mathrm{d}x = \operatorname{ar\,tanh} x$	Gains B1M0A0 (No integration undertaken)	
(b) **(i) & (ii)**	$f(x) = \operatorname{artanh} 2x - 8\arctan 4x$ $\Rightarrow \mathrm{f}'(x) = \dfrac{2}{1-4x^2} - \dfrac{32}{1+16x^2}$	<u>One term</u> correctly differentiated. Could be unsimplified	**B1**
	$1+16x^2 = 16-64x^2 \Rightarrow x^2 = \dfrac{3}{16} \Rightarrow x = \ldots$	Obtains a value for x from an equation of the form $\dfrac{a}{1-bx^2} - \dfrac{c}{1 \pm dx^2} = 0, \;\; a,b,c,d > 0$ i.e. accept sign errors with d only but allow recovery if recovered correctly	**M1**

	$x = \frac{\sqrt{3}}{4}$	$x = \frac{\sqrt{3}}{4}$ or exact equivalent. Not $\pm$ but no need to specifically reject the negative value	**A1**
	$y = \frac{1}{2}\ln\frac{\left(1+\frac{\sqrt{3}}{2}\right)}{\left(1-\frac{\sqrt{3}}{2}\right)} \left\{-8\arctan\sqrt{3}\right\}$	Uses correct logarithmic form of artanh with their value of x where $0 < x < \frac{1}{2}$	**M1**
	$y = \left\{\frac{1}{2}\ln\frac{\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)} - 8\arctan\sqrt{3}\right\} = \frac{1}{2}\ln\left(7+4\sqrt{3}\right) - \frac{8\pi}{3}$	Correct answer in the correct form	**A1**
			(5)
			Total 8