Question Number	Scheme	Notes	Marks
3(a)	$\det(\mathbf{M}-\lambda\mathbf{I}) = \begin{vmatrix} -2-\lambda & 1 & -3 \\ 0 & 3-\lambda & k \\ -2 & -2 & 3-\lambda \end{vmatrix} = (-2-\lambda)\lceil\lfloor(3-\lambda)(3-\lambda)+2k\rfloor\rceil - 1(0+2k) - 3\lceil\lfloor 2(3-\lambda)\rfloor\rceil$		
	Any recognisable attempt at $\det(\mathbf{M}-\lambda\mathbf{I})$. Allow recovery from invisible brackets May see e.g., $(-2-\lambda)\lceil\lfloor(3-\lambda)(3-\lambda)+2k\rfloor\rceil - 0 - 2\lceil\lfloor k+3(3-\lambda)\rfloor\rceil$ (1st column) Sarrus leads to $(-2-\lambda)(3-\lambda)(3-\lambda)-2k+0-\lceil\lfloor 6(3-\lambda)-2k(-2-\lambda)+0\rfloor\rceil$		**M1**
	$= (-2-\lambda)(9-6\lambda+\lambda^2+2k) - 2k - 18 + 6\lambda$ $= -\lambda^3+4\lambda^2+3\lambda-2k\lambda-4k-18-2k-18+6\lambda$ $\Rightarrow \lambda^3-4\lambda^2-3\lambda+2k\lambda+4k+18+2k+18-6\lambda=0$ $\Rightarrow \lambda^3-4\lambda^2+(2k-9)\lambda+6k+36=0$ **d**M1: Expands to a cubic and collects terms. No requirement for "$=0$" A1: Correct equation. Allow $+(2k-9)\lambda$ written as e.g., $-(9-2k)\lambda$ or $2k\lambda-9\lambda$		**dM1** **A1**
			(3)
(b)	$2^3-4\times2^2+2(2k-9)+6k+36=0\Rightarrow k=...$	Substitutes $\lambda=2$ into their equation and solves for k or uses long division to divide their equation in (a) by $\lambda-2$ and makes progress to achieve an equation in k (finding the remainder and setting equal to 0) Must use part (a) and any attempt not using (a) scores M0A0	**M1**
	$\Rightarrow 8-16-18+4k+6k+36=0$ $\Rightarrow 10+10k=0\Rightarrow k=-1$	Obtains $k=-1$ with no errors. Accept a clear verification using both values with minimal conclusion	**A1***
			(2)
(c)	$\{(-2)\times3\times(-\lambda)=6(-1)+36\Rightarrow 6\lambda=30\}$ $\{\text{or } \lambda^3-4\lambda^2-11\lambda+30=(\lambda-2)(\lambda+3)(\lambda-5)=0\}$ $\{\text{or trace = sum}\Rightarrow -2+3+3=2-3+\lambda\Rightarrow 4=-1+\lambda\}$ $\{\text{or } 2\times(-3)\times\lambda=\det\mathbf{M}=-30\Rightarrow 6\lambda=30\}$ $\lambda=5$ Correct eigenvalue		**B1**
			(1)
(d)	$\mathbf{Mx}=\lambda\mathbf{x}\Rightarrow \begin{matrix} -2x+y-3z=-3x \\ 3y-z=-3y \\ -2x-2y+3z=-3z \end{matrix}$ or $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=\mathbf{0}\Rightarrow \begin{matrix} x+y-3z=0 \\ 6y-z=0 \\ -2x-2y+6z=0 \end{matrix} \Rightarrow x=...,\ y=...,\ z=....$ Correct method to obtain simultaneous equations and solves for all variables to find values for each (at least one non-zero) Other approaches are possible, such as finding the cross product between any two rows of the $\mathbf{M}-\lambda\mathbf{I}$ matrix. If in doubt sent to review.		**M1**
	$\pm\begin{pmatrix}17\\1\\6\end{pmatrix}$	Correct eigenvector. Any integer multiple/either direction	**A1**
			(2)
			Total 8