3.

$$\mathbf{M} = \begin{pmatrix} -2 & 1 & -3 \\ 0 & 3 & k \\ -2 & -2 & 3 \end{pmatrix}$$

where k is a constant.

Given that λ is an eigenvalue of the matrix **M**

(a) show that

$$\lambda^3 - 4\lambda^2 + \mathrm{f}(k)\lambda + 6k + 36 = 0$$

where $\mathrm{f}(k)$ is a function to be determined.

(3)

Given that 2 is an eigenvalue of **M**

(b) use the answer to part (a) to show that $k = -1$

(2)

Given that -3 is also an eigenvalue of **M**

(c) write down the third eigenvalue of **M**

(1)

(d) Determine an eigenvector, with integer components, which corresponds to the eigenvalue -3

(2)

(Total for Question 3 is 8 marks)