Question Number	Scheme	Notes	Marks
	Note: any attempts that begin with the stated result in the question score 0		
4(a)	$y = \text{arsinh}\, x \Rightarrow \sinh y = x \Rightarrow \dfrac{e^{y} - e^{-y}}{2} = x$	Correct expression for x in exponentials	**B1**
	$\Rightarrow e^{2y} - 2xe^{y} - 1 = 0 \Rightarrow e^{y} = \dfrac{2x \pm \sqrt{4x^2+4}}{2}$ or $\left(e^{y} - x\right)^2 - x^2 - 1 = 0 \Rightarrow e^{y} = x \pm \sqrt{x^2+1}$	Obtains a 3TQ in e^{y} and attempts to solve by formula or CTS "= 0 " not required here. Must see either of these elements of working shown. Do not accept going straight to $e^{y} = x \pm \sqrt{x^2+1}$	**M1**
	$\left\{\ln\left(x\sqrt{\ \ x^2+1}\right) \text{is undefined since } x - \sqrt{x^2+1} < 0 \ \ \forall x\right\}$ $y = \ln\left(x + \sqrt{x^2+1}\right)$	Fully correct proof. Not $\pm$. May have just used + throughout. Reason for rejection of $-$ is not required and can be ignored.	**A1***
			(3)
ALT	$\underline{x = \text{arsinh}\, y} \Rightarrow \sinh x = y \Rightarrow \dfrac{e^{x} - e^{-x}}{2} = y$	The underlined statement must be seen (perhaps at the end) using this method to interchange the x and y. Correct expression for y in exponentials	**B1**
	$\Rightarrow e^{2x} - 2ye^{x} - 1 = 0 \Rightarrow e^{x} = \dfrac{2y \pm \sqrt{4y^2+4}}{2}$ or $\left(e^{x} - y\right)^2 - y^2 - 1 = 0 \Rightarrow e^{x} = y \pm \sqrt{y^2+1}$	Obtains a 3TQ in e^{x} and attempts to solve by formula or CTS (usual rules) "= 0 " not required here. Must see either of these elements of working shown. Do not accept going straight to $e^{x} = y \pm \sqrt{y^2+1}$	**M1**
	$\left\{\ln\left(y\sqrt{\ \ y^2+1}\right) \text{is undefined since } y - \sqrt{y^2+1} < 0 \ \ \forall y\right\}$ $\underline{x = \text{ar sinh}\, y = \ln\left(y + \sqrt{y^2+1}\right)}$ and so $\underline{\text{ar sinh}\, x = \ln\left(x + \sqrt{x^2+1}\right)}$	Fully correct proof. Not $\pm$. May have just used + throughout. Reason for rejection of $-$ is not required and can be ignored. There needs to be a clear connection between x and arsinh$y$ and a statement of the printed result, so the underlined statements are required	**A1***
			(3)
(b)	$y = (\sinh x - 3)\cosh x - 6x$ $\Rightarrow \dfrac{dy}{dx} = \sinh x(\sinh x - 3) + \cosh^2 x - 6$ or $\Rightarrow y = \dfrac{1}{2}\sinh 2x - 3\cosh x - 6x$ $\Rightarrow \dfrac{dy}{dx} = \cosh 2x - 3\sinh x - 6$	Differentiates to the correct form (condone coefficient errors). Allow if -6 is missing. If expanded first allow with $\sinh x \cosh x$ replaced with $k \sinh 2x$	**M1**
	$= \sinh^2 x - 3\sinh x + 1 + \sinh^2 x - 6$ or $= 2\sinh^2 x + 1 - 3\sinh x - 6$ $\left\{\Rightarrow 2\sinh^2 x - 3\sinh x - 5\right\}$	Uses $\cosh^2 x = \pm 1 \pm \sinh^2 x$ or $\cosh 2x = \pm 2\sinh^2 x \pm 1$ and obtains a quadratic expression in $\sinh x$	**M1**
	$2\sinh^2 x - 3\sinh x - 5 = 0$	Correct 3TQ = 0 (could be implied by correct values after solving, if working with inequalities etc throughout)	**A1**
	$(2\sinh x - 5)(\sinh x + 1) = 0$	Solves 3TQ by usual rules - no incorrect root if no working (could be implied by	**M1**

	$\sinh x = -1,\ \frac{5}{2}$	correct values after solving, if working with inequalities etc throughout)	
	$x = \ln\left(-1+\sqrt{(-1)^2+1}\right),\ x = \ln\left(\frac{5}{2}+\sqrt{\left(\frac{5}{2}\right)^2+1}\right)$	Either correct numerical expression	**A1**
	$\ln\left(\sqrt{2}-1\right) \leqslant x \leqslant \ln\left(\frac{5+\sqrt{29}}{2}\right)$ or $\ln\left(\sqrt{2}-1\right) < x < \ln\left(\frac{5+\sqrt{29}}{2}\right)$	Correct range in any suitable notation with simplified lns but allow $\ln\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)$ e.g. $x \in \left[\ln\left(\sqrt{2}-1\right), \ln\left(\frac{5+\sqrt{29}}{2}\right)\right]$ Allow with strict or non-strict inequalities, or soft/hard brackets	**A1**
	Working in exponentials: $\left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^x-e^{-x}}{2}-3\right)-6x$ $\Rightarrow \frac{\left(e^x+e^{-x}\right)\left(e^x-e^{-x}-6\right)}{4}-6x \Rightarrow \frac{e^{2x}-6e^x-6e^{-x}-e^{-2x}}{4}-6x \Rightarrow \frac{2e^{2x}-6e^x+6e^{-x}+2e^{-2x}}{4}-6$ $\Rightarrow e^{2x}-3e^x-12+3e^{-x}+e^{-2x}=0 \Rightarrow e^{4x}-3e^{3x}-12e^{2x}+3e^x+1=0$ Score second M1 for at least one hyperbolic function replaced correctly then 1st M1 for differentiating to a correct form ($e^{kx} \to ke^{kx}$ for each term) allowing the -6 to be missing. 1st A1 for a correct 5 term quartic equation in e^x Further marks require working with exact values so would require solving one of $e^{2x}-5e^x-1=0$ or $e^{2x}+2e^x-1=0$ by usual rules for the next M then last 2 marks as above		
			(6)
			Total 9