5. Relative to a fixed origin O, the points P, Q and R have coordinates

$$P(2, 5, 0) \quad Q(-2, 6, 3) \quad R(1, 2, -1)$$

(a) (i) Determine $\overrightarrow{OP} \times \overrightarrow{OQ}$

(ii) Hence determine the exact volume of the tetrahedron $OPQR$

(3)

Given that the plane Π_1 contains the points P, Q and R

(b) determine a Cartesian equation for Π_1

(4)

The points O, Q and R are in the plane Π_2 which has equation $\mathbf{r}.(12\mathbf{i} - \mathbf{j} + 10\mathbf{k}) = 0$

Given that the point S has coordinates $(-4, -3, 2)$

(c) determine the perpendicular distance from S to Π_2, giving the answer in the form $k\sqrt{5}$ where k is a fully simplified rational number.

(2)

(Total for Question 5 is 9 marks)

6.

$$\mathbf{A} = \begin{pmatrix} p & 3 & -1 \\ 0 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}$$

where p is a constant and $p \neq 3$

(a) Determine $\mathbf{A}^{-1}$, giving the answer in simplest form in terms of p.

(4)

The transformation represented by $\mathbf{A}$ transforms the point with position vector $2\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ to the point with position vector $16\mathbf{i} + 7\mathbf{j}$

(b) Show that $p = 4$

(1)

The transformation represented by $\mathbf{A}$ transforms the point B to the point with coordinates $(2, q, -1)$ where q is a constant.

(c) Use the answer to part (a) with $p = 4$ to determine the coordinates of B.
Give the answer in simplest form in terms of q.

(3)

(Total for Question 6 is 8 marks)