| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7(a)(i)** | $\frac{x^2}{45}+\frac{y^2}{20}=1\Rightarrow a^2=45,\ b^2=20$<br>$b^2=a^2\left(1-e^2\right)\Rightarrow 20=45\left(1-e^2\right)\Rightarrow e$ or $e^2=...$ | Uses a correct eccentricity formula with $a^2=45$ and $b^2=20$ and finds a value for $e$ or $e^2$ | **M1** |
| | $e=\frac{\sqrt{5}}{3}$ | Correct exact value. Any single fraction equivalent. Not $\pm$ | **A1** |
| **(a)(ii)** | $\frac{a}{e}=\frac{3\sqrt{5}}{"\frac{\sqrt{5}}{3}"}$ | For a <u>numerical</u> expression for $\frac{a}{e}$ with their $e$ but allow this mark if 45 is used for $a$ | **M1** |
| | $x=\pm 9$ | **Both** correct equations.<br>$x=\pm 9$ or $x=9,\ x=-9$ only | **A1** |
| | | | **(4)** |
| **(b)** | $x=2\cosh 2t,\ y=8\sinh t$<br>$\Rightarrow x=2\left(2\sinh^2 t+1\right)$ | Replaces $\cosh 2t$ with $\pm 2\sinh^2 t\pm 1$ oe<br>Missing '$h$'s is M0 but if recovered A0* | **M1** |
| | $\frac{y^2}{64}=\sinh^2 t\Rightarrow x=2\left(\frac{2y^2}{64}+1\right)$ | Proceeds to eliminate $t$ | **dM1** |
| | $x=\frac{y^2}{16}+2\Rightarrow y^2=16x-32$ * | Correct equation with no errors. Any notational slips will be penalised for this mark, such as missing '$h$' or '$x$' instead of '$t$' but condone siht for sinht etc | **A1*** |
| | | | **(3)** |
| **ALT 1** | $y=8\sinh t=8\left(\frac{e^t-e^{-t}}{2}\right)$<br>$y^2=64\sinh^2 t=64\left(\frac{e^{2t}-2+e^{-2t}}{4}\right)$ | Replaces $y$ with the correct exponential form and squares | **M1** |
| | $=16\left(e^{2t}+e^{-2t}\right)-32$<br>$=16\left(2\cosh 2t\right)-32$ | Separates showing the correct exponential form for $\cosh 2t$ | **dM1** |
| | $y^2=16x-32$ | Correct equation with no errors | **A1** |
| | | | **(3)** |
| **ALT 2** | $x=2\cosh 2t,\ y=8\sinh t$<br>$\Rightarrow x=2\left(2\cosh^2 t-1\right)$ | Replaces $\cosh 2t$ with $\pm 2\cosh^2 t\pm 1$ oe<br>Missing '$h$'s is M0 but if recovered A0* | **M1** |
| | $\cosh^2 t-\sinh^2 t=1$<br>$\Rightarrow\left(\frac{x+2}{4}\right)-\left(\frac{y}{8}\right)^2=1$ | Proceeds to eliminate $t$ | **dM1** |
| | $16\left(x+2\right)-y^2=64$<br>$\Rightarrow y^2=16x-32$ | Correct equation with no errors. Any notational slips will be penalised for this mark, such as missing '$h$' or '$x$' instead of '$t$' but condone siht for sinht etc | **A1*** |
| | | | **(3)** |
| **(c)** | $\frac{x^2}{45}+\frac{16x-32}{20}=1\Rightarrow 20x^2+45\left(16x-32\right)=900$<br>$20x^2+720x-2340=0\Rightarrow x^2+36x-117=0$ | Substitutes $y^2=16x-32$ into $E$ and solves a resulting 3TQ (usual rules - no incorrect root if no working) | **M1** |

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| | $\Rightarrow x = ...$ | | |
| | $\{(x-3)(x+39)=0 \Rightarrow\} x = 3$ | Correct value | **A1** |
| | $2y\dfrac{dy}{dx} = 16$ or $\dfrac{dy}{dx} = \dfrac{1}{2}(16x-32)^{-\frac{1}{2}}(16)$<br>**OR**<br>$\dfrac{dx}{dt} = 4\sinh 2t$ and $\dfrac{dy}{dt} = 8\cosh t$ | Correct expression involving $\dfrac{dy}{dx}$<br>**OR**<br>Correct expressions for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$<br>Allow if the derivatives are seen in (b) | **B1** |

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| | $2\pi\int y\sqrt{1+\dfrac{64}{y^2}\dfrac{dx}{dy}}\,dy$ or $2\pi\int(16x-32)^{\frac{1}{2}}\sqrt{1+\dfrac{64}{16x-32}}\,dx$<br>Or $2\pi\int 8\sinh t\sqrt{16\sinh^2 2t + 64\cosh^2 t}\,dt$<br>Uses $...\int y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\{dx\}$ OR $...\int y\sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}\{dx\}$<br>There must be some attempt to apply this to the question with substitutions seen, not simply stated. The integral must be in terms of one variable but ignore the limits (if given) for this mark. Ignore error with (or missing) $2\pi$<br>Integral may be seen later | | **M1** |
| | $= 2\pi\int(16x-32+64)^{\frac{1}{2}}\,dx$ or e.g., $8\pi\int(x+2)^{\frac{1}{2}}\,dx$<br>Or<br>$2\pi\int 8\sinh t\sqrt{64\cosh^2 t\left(1+\sinh^2 t\right)}\,dt = 2\pi\int 8\sinh t 8\cosh^2 t\,dt$<br>Or<br>$= \dfrac{\pi}{4}\int y\left(y^2+64\right)^{\frac{1}{2}}\,dy$ | Correct and in integrable form | **A1** |
| | $= \dfrac{16\pi}{3}(x+2)^{\frac{3}{2}}$<br>Or<br>$= \dfrac{128\pi}{3}\cosh^3 t$<br>Or<br>$= \dfrac{\pi}{12}\left(y^2+64\right)^{\frac{3}{2}}$ | For $(Cx+D)^{\frac{1}{2}} \rightarrow \dfrac{(Cx+D)^{\frac{3}{2}}}{\frac{3}{2}}$<br>Or<br>$a\sinh t\ b\cosh^2 t\,dt \rightarrow k\cosh^3 t$<br>Or<br>For $y\left(y^2+C\right)^{\frac{1}{2}} \rightarrow \dfrac{\left(y^2+C\right)^{\frac{3}{2}}}{\frac{3}{2}}$ | **M1** |

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| | $\left[\frac{16\pi}{3}(x+2)^{\frac{3}{2}}\right]_3^9 = \frac{16\pi}{3}\left(11\sqrt{11}-5\sqrt{5}\right)$ | Correct answer in correct form. Use of parametric form is unlikely to give an exact answer in the form required, as the limits are $t_1 = \frac{1}{2}\ln\left(\frac{3+\sqrt{5}}{2}\right)$ and $t_2 = \frac{1}{2}\ln\left(\frac{9+\sqrt{77}}{2}\right)$ If in doubt send to review | **A1** |
| | | | **(7)** |
| | | | **Total 14** |