8. $$y = \sin^{n-1} x \cos x$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (n-1)\sin^{n-2} x - n\sin^{n} x$$

(2)

$$I_n = \int \mathrm{e}^{6x} \sin^n x \, \mathrm{d}x \qquad n \geqslant 0$$

(b) Use integration by parts to show that, for $n \geqslant 2$

$$(n^2 + 36)I_n = \mathrm{e}^{6x} \sin^{n-1} x(6\sin x - n\cos x) + n(n-1)I_{n-2}$$

(5)

(c) Hence show that

$$\int_0^{\frac{\pi}{6}} \mathrm{e}^{6x} \sin^2 x \, \mathrm{d}x = \frac{1}{240}\left(\mathrm{e}^{\pi}\left(p + q\sqrt{3}\right) + r\right)$$

where p, q and r are integers to be determined.

(4)

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS