4. In this question you may assume, without proof, that the distance of the centre of mass of a uniform semicircular lamina of radius r from its diameter is $\frac{4r}{3\pi}$

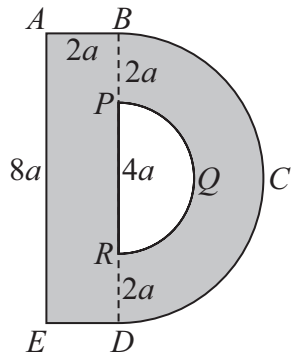


Figure 2

The uniform lamina $ABDE$ is a rectangle with $AB = 2a$ and $AE = 8a$.

The uniform lamina BCD is a semicircle with diameter $BD = 8a$.

The points P and R lie on BD with $BP = RD = 2a$.

The uniform lamina PQR is a semicircle with diameter $PR = 4a$.

The mass per unit area of the lamina $ABDE$ is equal to the mass per unit area of the lamina BCD.

The uniform template $ABCDE$, shown shaded in Figure 2, is formed by joining the lamina $ABDE$ to the lamina BCD and removing the lamina PQR.

The distance of the centre of mass of the template **from *BD*** is d.

(a) Show that $d = \frac{32a}{3(8+3\pi)}$

(5)

The template is free to rotate in a vertical plane about a smooth horizontal axis through B and hangs freely in equilibrium with BD at an angle $\phi°$ to the downward vertical.

(b) Find the value of ϕ.

(3)

The weight of the template is $15W$. A particle of weight kW is attached to the template at E.

The template is again free to rotate in a vertical plane about a smooth horizontal axis through B. The template is held in equilibrium, with BD vertical, by a horizontal force of magnitude W which is applied to the template at E and acts towards D.

(c) Find the value of k.

(4)

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Question 4 continued

(Total for Question 4 is 12 marks)