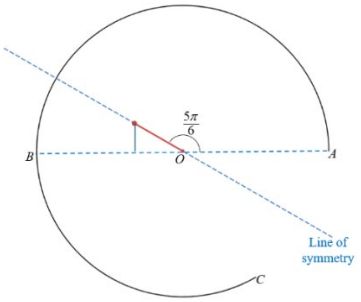
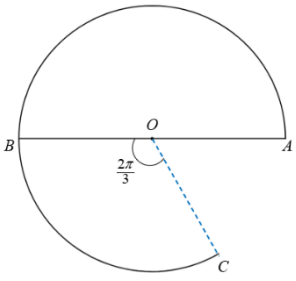
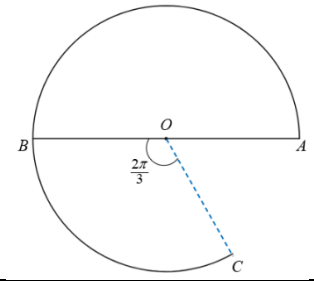
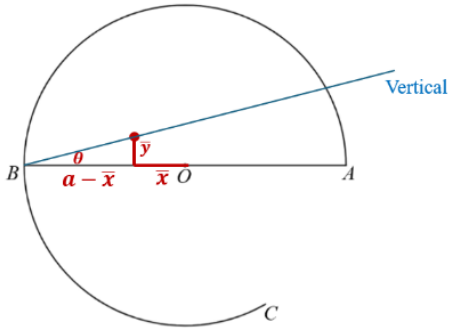
Question Number	Answer	Mark
	N.B. If areas are involved there are no marks available.	
4(a)	Using arc ABC + BA	
	For arc ABC, distance of centre of mass from O  $d = \dfrac{a\sin\left(\frac{5\pi}{6}\right)}{\frac{5\pi}{6}} = \dfrac{3a}{5\pi}$	B1
	<table><tr><td></td><td>Arc ABC</td><td>Line BA</td><td>Total</td></tr><tr><td>Mass ratio</td><td>$\frac{5\pi a}{3}$</td><td>$2a$</td><td>$a\left(\frac{5\pi}{3}+2\right)$</td></tr><tr><td>Distance from BA</td><td>$\frac{3a}{10\pi}$</td><td>0</td><td>$\bar{y}$</td></tr></table>	B1 mass ratios B1 dist
	Moments about BA or a parallel axis	M1
	M(BA): $\dfrac{5\pi a}{3}\times\dfrac{3a}{10\pi} = a\left(\dfrac{5\pi}{3}+2\right)\bar{y}$	A1
	$\bar{y} = \dfrac{3a}{2(5\pi+6)}$ *	A1*
		(6)
(a) ALT1	Using arc AB + BA + arc BC	
	 For arc BC, distance of centre of mass from O $= \dfrac{a\sin\left(\frac{\pi}{3}\right)}{\frac{\pi}{3}} = \dfrac{3\sqrt{3}a}{2\pi}$ **OR** for arc AB, distance of centre of mass from O $= \dfrac{a\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \dfrac{2a}{\pi}$	B1

	<table><tr><td></td><td>Arc AB</td><td>Line BA</td><td>Arc BC</td><td>Total</td></tr><tr><td>Mass ratio</td><td>πa</td><td>$2a$</td><td>$\frac{2\pi a}{3}$</td><td>$a\left(\frac{5\pi}{3}+2\right)$</td></tr><tr><td>Distance From BA</td><td>$\frac{2a}{\pi}$</td><td>0</td><td>$\frac{9a}{4\pi}$</td><td>$\bar{y}$</td></tr></table>	B1 mass ratios B1 dist
	Moments about BA or a parallel axis	M1
	M(BA): $\pi a \times \frac{2a}{\pi} - \frac{2\pi a}{3} \times \frac{9a}{4\pi} = a\left(\frac{5\pi}{3}+2\right)\bar{y}$	A1
	$\bar{y} = \frac{3a}{2(5\pi+6)}$ *	A1*
		(6)
(a) ALT 2	Using Circle $+ BA -$ arc AC	
	For arc AC, distance of centre of mass from O  $= \frac{a\sin\left(\frac{\pi}{6}\right)}{\frac{\pi}{6}} = \frac{3a}{\pi}$	B1
	<table><tr><td></td><td>Circle</td><td>Line BA</td><td>Arc AC</td><td>Total</td></tr><tr><td>Mass ratio</td><td>$2\pi a$</td><td>$2a$</td><td>$\frac{\pi a}{3}$</td><td>$2\pi a + 2a - \frac{\pi a}{3}$</td></tr><tr><td>Distance from BA</td><td>0</td><td>0</td><td>$\frac{3a}{2\pi}$</td><td>$\bar{y}$</td></tr></table>	B1 mass ratios B1 dist
	Moments about BA or a parallel axis	M1
	M(BA): $0+0-(-\frac{\pi a}{3}\times\frac{3a}{2\pi}) = \left(2\pi a + 2a - \frac{\pi a}{3}\right)\bar{y}$	A1
	$\bar{y} = \frac{3a}{2(5\pi+6)}$ *	A1*
	S.C. Moments about an axis through O, perpendicular to the line of symmetry and multiply $\bar{y}$ by $\sin\frac{\pi}{6}$ $0+0-\left(-\frac{\pi a}{3}\times\frac{3a}{\pi}\right) = \left(2\pi a + 2a - \frac{\pi a}{3}\right)\bar{y}$	M1 A1

	: $\bar{y} = \dfrac{3a}{2(5\pi+6)}$ *	A1*
		(6)
4(b)		
	M(axis through O): (using arc ABC + BA) $\dfrac{5\pi a}{3} \times \left(\dfrac{3a}{5\pi} \times \dfrac{\sqrt{3}}{2} \right) = a\left(\dfrac{5\pi}{3} + 2 \right)\bar{x}$ **OR:** M(axis through O): (using circle + BA − AC) $0 + 0 - (-\dfrac{\pi a}{3} \times \dfrac{3a\sqrt{3}}{2\pi}) = \left(2\pi a + 2a - \dfrac{\pi a}{3} \right)\bar{x}$ **OR:** M(axis through O): (arc AB + BA + arc BC) $(\pi a \times 0) + (2a \times 0) + \dfrac{2\pi a}{3} \times \dfrac{3a\sqrt{3}}{4\pi}) = \left(\pi a + 2a + \dfrac{2\pi a}{3} \right)\bar{x}$ **OR:** Using symmetry: $\bar{x} = \bar{y}\tan\dfrac{\pi}{3} = \dfrac{3a\sqrt{3}}{10\pi+12}$	M1 A1 M1 A1 M1 A1 M1 A1
	Method to find a relevant angle  $\tan\theta = \dfrac{\bar{y}}{a-\bar{x}}$ $= \dfrac{\dfrac{3a}{2(5\pi+6)}}{a - \left(\dfrac{3a\sqrt{3}}{10\pi+12} \right)}$ $\left(= \dfrac{3}{10\pi+12-3\sqrt{3}} \right)$	M1 A1
	$\theta = 4.5$ 1 d.p.	A1
		(5)
4(b) ALT 1	M(axis through B) to find $\bar{x}$	M1
	(using arc ABC + BA): $\dfrac{5\pi a}{3} \times \left(a - \left(\dfrac{3a}{5\pi} \times \dfrac{\sqrt{3}}{2} \right) \right) + (2a \times a) = \left(\dfrac{5\pi a}{3} + 2a \right)\bar{x}$	

	OR: (using arc AB + BA + arc BC) $(\pi a \times a) + (2a \times a) + \frac{2\pi a}{3} \times \left(a - \frac{3a\sqrt{3}}{4\pi} \right) = \left(\frac{5\pi a}{3} + 2a \right) \bar{x}$ **OR:** (using circle + BA − AC) $(2\pi a \times a) + (2a \times a) - \frac{\pi a}{3} \times \left(a + \frac{3a\sqrt{3}}{2\pi} \right) = \left(\frac{5\pi a}{3} + 2a \right)$	A1
	$\tan\theta = \frac{\bar{y}}{\bar{x}}$ $= \frac{\frac{3a}{2(5\pi+6)}}{a\left(\frac{10\pi+12-3\sqrt{3}}{10\pi+12} \right)}$ $= \frac{3}{10\pi+12-3\sqrt{3}}$	M1 A1
	$\theta = 4.5$ 1 d.p.	A1
		(5)
		(11)
	Notes for question	
(a)		
B1	Correct distance from O seen or implied for an arc, $\frac{r\sin\alpha}{\alpha}$ where $\alpha = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$	
B1	Correct mass ratios, including total mass.	
B1	Correct distances. Allow $\pm$.	
M1	Moments about BA or a parallel axis. Condone sin/cos confusion. Must be dimensionally consistent and distances must be resolved **and correct no. of terms in the total mass**	
A1	Correct unsimplified equation.	
A1*	Obtain given answer from complete and correct working. Must see at least one line of working or simplification before reaching the given answer.	
(a) ALT1		
B1	Correct distance from O seen or implied for an arc, $\frac{r\sin\alpha}{\alpha}$ where $\alpha = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$	
B1	Correct mass ratios	
B1	Correct distances	
M1	Moments about BA or a parallel axis. Condone sin/cos confusion. Must be dimensionally consistent.	
A1	Correct unsimplified equation.	

A1*	Obtain given answer from complete and correct working. Must see at least one line of working or simplification before reaching the given answer.	
(b)		
M1	Moments equation about axis perpendicular to *BA* to find $\overline{x}$ with correct structure. Must be dimensionally consistent.and distances must be resolved **and correct no. of terms in the total mass.**	
A1	Correct unsimplified equation.	
M1	Complete method using trig to find an equation in θ. allow $\tan\theta = \frac{-\overline{y}}{a-\overline{x}}$ or $\tan\theta = \frac{-\overline{y}}{\overline{x}}$ as appropriate for their $\overline{x}$ and $\overline{y}$ but must be dimensionally consistent. **N.B.** $\overline{x}$ and $\overline{y}$ do NOT need to be substituted for this mark.	
A1	Correct unsimplified equation	
A1	Correct answer to 1dp (**N.B.** 4.488138..is the unrounded value)	