| Question Number | Scheme | Marks |
|---|---|---|
| **5(a)** | $(\pi\rho)\int_0^r xy^2\,\mathrm{d}x$ | |
| | $=(\pi\rho)\int_0^r x\left(r^2-x^2\right)\mathrm{d}x$ | M1 |
| | $=(\pi\rho)\left[\frac{1}{2}x^2r^2-\frac{x^4}{4}\right]_0^r$ | A1 |
| | $=(\pi\rho)\frac{r^4}{4}$ | A1 |
| | $\frac{2\pi\rho r^3}{3}\bar{x}=\pi\rho\int xy^2\,\mathrm{d}x$ | M1 |
| | $\bar{x}=\frac{\pi\rho r^4}{4}\div\frac{2\pi\rho r^3}{3}=\frac{3}{8}r$ * | A1* (5) |
| **(b)** | Hemisphere Cone | |
| | Mass $\frac{2}{3}\pi r^3$ $\frac{1}{3}\pi kr^3$ | B1 |
| | Dist of c of m from centre of common plane $\frac{3}{8}r$ $\frac{1}{4}kr$ | B1 |
| | $\frac{2}{3}\times\frac{3}{8}r=\frac{k}{3}\times\frac{1}{4}kr$ | M1A1ft |
| | $k^2=3\quad k=\sqrt{3}$ | A1 (5) [10] |

**(a)**

**M1** Use of $(\pi\rho)\int_0^r xy^2\,\mathrm{d}x$ with $y^2=r^2-x^2$ and attempt the integration. Limits not needed.

**A1** Correct integration – limits not needed

**A1** Sub correct (upper) limit. (Sub of 0 not needed)

**M1** Use of $V\rho\bar{x}=\pi\rho\int xy^2\,\mathrm{d}x$ with their result to obtain $\bar{x}=\ldots$ where $V$ is the volume of the hemisphere or sphere ($\pi$ , $p$ must be on both sides or neither)

**A1*** $\bar{x}=\frac{3}{8}r$

**(b)**

**B1** Correct mass ratio for hemisphere and cone. Total mass not needed for this mark.

**B1** Correct distances of c of m for cone and hemisphere from centre of common plane (or another point). Both can be positive or one can be negative.

Distances from vertex of cone (H) $kr+\frac{3}{8}r$ (C) $\frac{3}{4}kr$

Distances from peak of hemisphere (H) $\frac{5}{8}r$ (C) $r+\frac{1}{4}kr$

**M1** Form a dimensionally correct moments equation **with the correct value for $\bar{x}$ depending on where they have taken moments**. (0 from plane face, $kr$ from vertex of cone, $r$ from peak of hemisphere) Allow even if formula for sphere is used. Ignore signs.

| | |
|---|---|
| **A1ft**<br>**A1** | Correct equation, follow through their masses and distances, signs to be correct here.<br>Correct exact result. |