3.

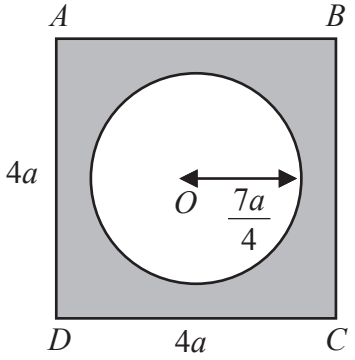


Figure 3

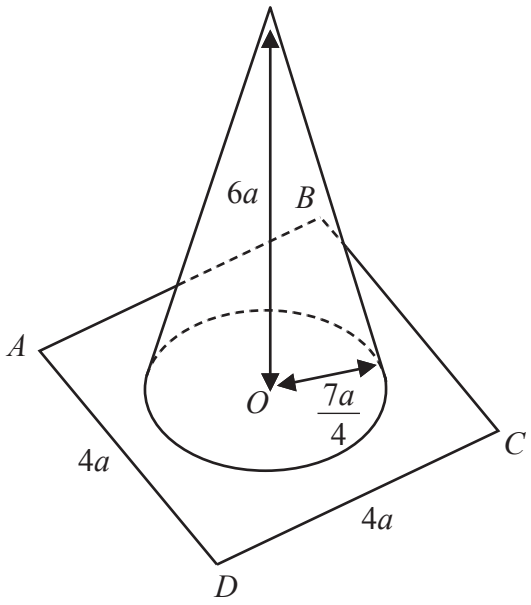


Figure 4

A square $ABCD$ of side $4a$ is made from thin uniform cardboard. The centre of the square is O. A circle with centre O and radius $\frac{7a}{4}$ is then removed from the square to form a template T, shown shaded in Figure 3.

A right conical shell, with no base, has radius $\frac{7a}{4}$ and perpendicular height $6a$.

The shell is made of the same thin uniform cardboard as T.

The shell is attached to T so that the circumference of the end of the shell coincides with the circumference of the circle centre O, to form the hat H, shown in Figure 4.

[*The surface area of a right conical shell of radius r and slant height l is* πrl.]

(a) Show that the exact distance of the centre of mass of H from O is

$$\frac{175\pi a}{(63\pi + 128)}$$

(8)

A fixed rough plane is inclined to the horizontal at an angle α. The hat H is placed on the plane, with $ABCD$ in contact with the plane, and AB parallel to a line of greatest slope of the plane. The plane is sufficiently rough to prevent the hat from sliding down the plane.

Given that the hat is on the point of toppling,

(b) find the exact value of $\tan \alpha$, giving your answer in simplest form.

(2)

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Question 3 continued

(Total for Question 3 is 10 marks)