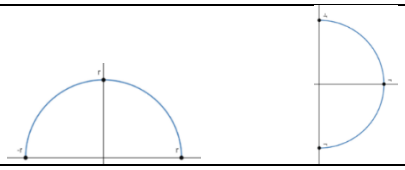
5			Curve equation $x^2 + y^2 = r^2$
5a	Using *x*-axis $(\rho)\int x \times 2\sqrt{r^2 - x^2}\,\mathrm{d}x$ or Using *y*-axis $(\rho)\frac{1}{2}\int 2\left(\sqrt{r^2 - x^2}\right)^2 \mathrm{d}x$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: *x*-axis: $k\int x\sqrt{r^2 - x^2}\,\mathrm{d}x$ *y*-axis: $k\int r^2 - x^2\,\mathrm{d}x$
	x-axis $= -\frac{2}{3}(\rho)\left(r^2 - x^2\right)^{\frac{3}{2}}$ *y*-axis $= (\rho)\left(xr^2 - \frac{x^3}{3}\right)$	A1	Correct integration, ignore limits. Correct expression.
	$= \frac{2}{3}(\rho)r^3$	A1	Correct use of limits, 0 and *r* or – *r* and *r*.
	Using *x*-axis $\frac{1}{2}\pi r^2 \rho \bar{x} = \rho\int_0^r 2xy\,\mathrm{d}x$ Using *y*-axis $\frac{1}{2}\pi r^2 \rho \bar{y} = \rho\frac{1}{2}\int_{-r}^{r} y^2\,\mathrm{d}x$ or $\frac{1}{2}\pi r^2 \rho \bar{y} = \rho\int_0^r y^2\,\mathrm{d}x$	M1	Complete method to obtain distance. Use of a correct formula, consistent with the axis and limits used, to find centre of mass with curve equation. ρ must appear on both sides or neither.
	$\bar{x} = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2} = \frac{4r}{3\pi}$ *	A1*	Obtain given answer from correct working
ALT 1 5(a)	Parametric approach $x = r\cos\theta,\ \ y = r\sin\theta$		Curve equation $x^2 + y^2 = r^2$
	Using *x*-axis $2r^3\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta\,\mathrm{d}\theta$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: $kr^3\int \sin^2\theta\cos\theta\,\mathrm{d}\theta$

	$= 2r^3\left[\dfrac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{2}}$	A1	Correct integration, ignore limits. Correct expression.
	$= \dfrac{2}{3}r^3$	A1	Correct use of limits
	$\bar{x} = \dfrac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2}$	M1	Complete method to obtain distance. Use of correct formula. ρ must appear on both sides or neither.
	$= \dfrac{4r}{3\pi}$ *	A1*	Obtain given answer from correct working
		[5]	
ALT 2 5(a)	Using *y*-axis $r^3\int_0^{\frac{\pi}{2}}\sin^3\theta\ \mathrm{d}\theta$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: $kr^3\int\sin^3\theta\,\mathrm{d}\theta$
	$r^3\int_0^{\frac{\pi}{2}}\left(1-\cos^2\theta\right)\sin\theta\ \mathrm{d}\theta$ $= r^3\left[-\cos\theta+\dfrac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$	A1	Correct integration, ignore limits. Correct expression.
	$= \dfrac{2}{3}r^3$	A1	Correct use of limits
	$\bar{x} = \dfrac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2}$	M1	Complete method to obtain distance. Use of correct formula. ρ must appear on both sides or neither.
	$= \dfrac{4r}{3\pi}$ *	A1*	Obtain given answer from correct working
		[5]	
5b	<table><tr><td></td><td>large</td><td>Small removed</td><td>Small added</td></tr><tr><td>mass</td><td>$8\pi a^2$</td><td>$2\pi a^2$</td><td>$2\pi a^2$</td></tr><tr><td>From *AC*</td><td>$\dfrac{16a}{3\pi}$</td><td>$\dfrac{8a}{3\pi}$</td><td>$(-)\dfrac{8a}{3\pi}$</td></tr></table>	B1 B1	Correct mass ratios Correct distances
	Moments about *AC*	M1	All terms required. Dimensionally correct or equivalent for a parallel axis. Condone sign errors. If column vectors are used, this mark is awarded once the equation is written separate to the column vectors.

	$8\pi a^2 \times \frac{16a}{3\pi} - 2\pi a^2 \times \frac{8a}{3\pi} - 2\pi a^2 \times \frac{8a}{3\pi} = 8\pi a^2 d$	A1	Correct unsimplified equation.
	$\frac{96a}{3\pi} = 8d \Rightarrow d = \frac{4a}{\pi}$ *	A1*	Obtain given value from correct working. Need to see at least some simplification.
		[5]	
5c	Moments about perpendicular axis through *A*	M1	Dimensionally correct. Need all terms. Or equivalent for a parallel axis
	From *A* $4a \times 8\pi a^2 - 2a \times 2\pi a^2 + 6a \times 2\pi a^2 = 8\pi a^2 \bar{x}$	A1ft A1ft	Unsimplified equation with at most one error. Correct unsimplified equation Follow their mass ratio
	$\Rightarrow \bar{x} = 5a$	A1	Correct only. If measured from *B*, distance is *a*
	Correct use of trig to find an expression for $\tan\theta$	M1	$\tan\theta = \frac{d}{\bar{x}}$ or $\tan\theta = \frac{\bar{x}}{d}$ where $\bar{x}$ is distance from *A*.
	$\tan\theta = \frac{4}{5\pi}$	A1	Only
		[6]	
		(16)	