5. (a) Use algebraic integration to show that the centre of mass of a uniform semicircular disc of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from the diameter through O

[You may assume, without proof, that the area of a circle of radius r is πr^2]

(5)

A uniform lamina L is in the shape of a semicircle with centre B and diameter $AC = 8a$. The semicircle with diameter AB is removed from L and attached to the straight edge BC to form the template T, shown shaded in Figure 4.

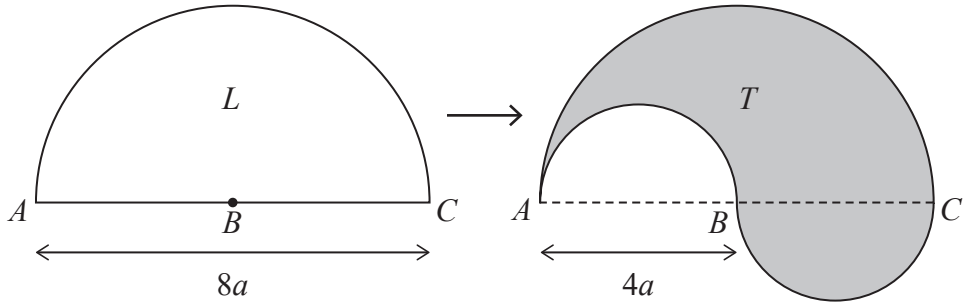


Figure 4

The distance of the centre of mass of T from AC is d.

(b) Show that $d = \frac{4a}{\pi}$

(5)

The template T is freely suspended from A and hangs in equilibrium with AC at an angle θ to the downward vertical.

(c) Find the exact value of $\tan\theta$

(6)

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Question 5 continued

(Total for Question 5 is 16 marks)