5. A uniform right solid circular cone C has radius r and height $4r$.

(a) Show, using algebraic integration, that the distance of the centre of mass of C from its vertex is $3r$.

[You may assume that the volume of C is $\frac{4}{3}\pi r^3$]

(4)

A uniform solid S, shown below in Figure 3, is formed by **removing** from C a uniform solid right circular cylinder of height r and radius $\frac{1}{2}r$, where the centre of one end of the cylinder coincides with the centre of the plane face of C and the axis of the cylinder coincides with the axis of C.

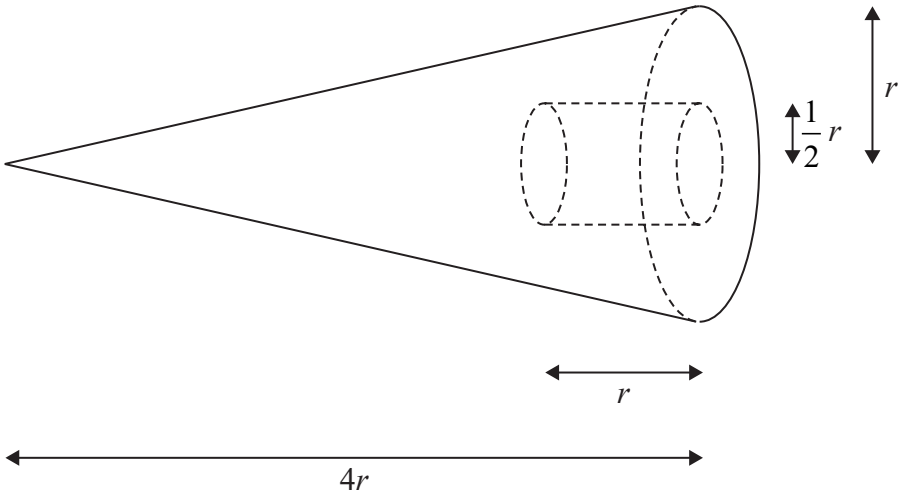


Figure 3

(b) Show that the distance of the centre of mass of S from the vertex of C is $\frac{75}{26}r$

(5)

A rough plane is inclined at an angle α to the horizontal.

The solid S rests in equilibrium with its plane face in contact with the inclined plane.

Given that S is on the point of toppling,

(c) find the exact value of $\tan\alpha$

(3)

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Question 5 continued

(Total for Question 5 is 12 marks)